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KEPLER AND GRAVITATION

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Some aspects of Kepler's contribution to the explanation of the law of gravitation are discussed with a historical background. The transition from Keplerian to Newtonian ideas is explained.

Kepler's contribution to the discovery of the universal law of gravitation rests first of all on the formulation of his three laws, which enshrine an explicit solution for the one-body problem in celestial mechanics. If we take into account Newton's Law of reaction, replace the Sun by the centre of gravity of the Sun-Planet system, and its mass M by $(M + m)$, the sum of the solar and planetary masses, Kepler's Laws also give the solution of the two-body problem; thus Newton's theory of gravitation as such only leads to new insights for the n-body problem.

In his *Astronomia Nova* Kepler first treated the planetary motions entirely kinematically, following the classical methods used from Ptolemy's time to that of the Copernicus; the major difference was that Kepler abandoned the Platonic axioms of motion, as well as the idea of heliocentricity. Thus, he made no *a priori* assumptions about the shape of the orbits. The *Astronomia Nova*, and even more the *Harmonice Mundi*, through their statements of Kepler's Second and Third Laws implicitly contained the inverse square law for the force between Sun and Planet. Commenting on this, some philosophers (Hegel, Schopenhauer) and physicists have declared Newton's theory of gravitation to be "no more than Kepler's Laws differentiated twice with respect to time" (Drude). This view is incorrect, for Kepler's Laws lead only to the solution of the one- and two-body problems, while Newton's theory, in principle at least, sets outs the gravitational forces between any number of bodies, and thus lays a foundation for the treatment of the *n*-body problem, as well as for perturbation theory of equilibrium, tidal theory, gravimetry, and much else.

Seen from the standpoint of Newton's gravitational mechanics, Kepler's Laws yield as much information for the two-body problem as the first ten general integrals of the equations of motion in a Newtonian gravity field (excepting trivial solutions easily rejected on grounds of symmetry). These results just suffice for a unique solution of the two-body problem: on the other hand, a synthesis based on the general equations of motion does not lead to a complete solution for the *n*-body problem. Thus, the move from Kepler's synthetic-kinetic approach to Newton's analytic-dynamic system of mechanics was essential.

Kepler himself - and this is his second contribution to the development of gravitation theory - always called for such a dynamical treatment, but was himself only able to give a qualitative idea of one. Even so, both the *Astronomia Nova* and the second edition of the *Mysterium Cosmographicum* contain the concepts of a universal force of attraction between all heavenly bodies - on the lines of William Gilbert's magnetism (1600) - as well as at least a qualitative expression for the falling-off of the force with distance. Thus, the Moon's motion around the Earth led Kepler to the conclusion that not only the Sun but also the planets were centers for a force field of this nature - a field analogous to a magnetic field. However, it seems that Kepler did not reach the ultimate conclusion that all bodies possess a gravitational field, and that its strength is proportional to the body's mass.

It is also noteworthy that over the years Kepler moved from a mystical, Phythagorean approach to a quasi-mechanical theory of the origin of gravitational forces. His concept of a kinematic origin of gravity as resulting from "moving fluids" surrounding the Sun and the planets, which seek to draw the bodies directly to the center of mass, is the prototype of all mechanical theories of gravitation, from Descartes through Newton to Lesage. At least qualitatively, Kepler here grasped Newton's fundamental insight that the gravity we observe on Earth and the force that controls the planetary motions are of the same nature.

The integral treatment of motions in a gravitational field, as used by Kepler (instead of an explicit gravitational dynamics), is reflected to some extent in Einstein's first treatment of the one-body problem on the basis of the geodetic law of motion, in the General Theory of Relativity. Einstein's postulate that the motion of a planet about a central body is described by a geodetic world-line in the spherical-symmetrical Schwarzschild-metric is a quite general statement. The spherical symmetry then of itself implies Kepler's Second Law, while the Third Law is approximately contained in the definition of relative energy in the Schwarzschild space. We know that the relativistic one-body problem does not lead to a simple curve as the path of motion, but to elliptical integrals, which approximate to Keplerian ellipses with advancing perihelia.

The two-body problem cannot be treated analytically in the General Theory of Relativity. A solution to any desired accuracy is possible since an integral treatment can be derived by the geodetic principle, using the Einstein field equations and the special conditions describing the singularities, i. e. the sources of gravitational fields (methods of

Einstein, Infeld and Hoffmann, and also Fock). Here, too, the first approximation leads to Keplerian orbits about the Newtonian centre of mass, while the second approximation gives a perihelion motion dependent on the total mass of the two bodies.

Kepler's integral treatment of gravitational dynamics was the first historical step toward Newton's analytical formulation of the differential equations of motion and of the Law of Reaction. Since Newton's theory involves action at a distance, the introduction of an explicit Law of Reaction is equivalent to the later formulation by Laplace and Poisson of field (potential) equations for the gravitational field. Only with Einstein's gravitational theory do we see a true gravitational field theory, which involves more in principle than can be derived from reaction terms in the equations of motion.

Kepler's synthetic treatment of problems of motion acquires new significance for quite different types of force interactions. In nuclear physics it has not so far been possible to set up a consistent quantum theory for the strong interactive forces between elementary particles, and the *ad hoc* use of specific reaction terms is largely arbitrary and inconsistent. Hence, the fruitful treatment of the scattering, etc., of elementary particles is based on the integration of the general equations of motion and the additional conditions of symmetry, which govern the dynamics of the scattered particles. This is a modern analogue of Kepler's synthetic treatment of planetary motions, and it, too, meets with all the same difficulties with regard to the uniqueness of the treatment of the many-body problem.

We cannot yet say whether the step to Newtonian type laws of motion (which must here of necessity be relativistic and involve field theory) is actually possible in elementary particle theory. Possibly, too, the transition from Keplerian to Newtonian treatments will be tried to specified types of reactions, such as gravitation or electromagnetic forces.