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KERR GEOMETRY X.

COVARIANT DECOMPOSITION OF THE KERR-NEWMAN FIELD EQUATIONS

Rainer Burghardt*

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Contents

We re-arrange the metric and the field equations of the Kerr-Newman model in such a way that a [3+1]-decomposition of the field equations can easily be performed. We obtain gravielectric equations with gravitational and electric field currents and field energy. We explain the geometry by embedding it into a higher dimensional flat space. We work out the structure of the embedded surfaces and we show how the rotation is invoked into the theory.

* e-mail: arg@aon.at, home page: http://arg.or.at/

1. INTRODUCTION

Newman et al. [1] generalized the charged Reissner-Nordström metric [2] to the metric of a rotating system. It turned out that this metric is closely related to the Kerr metric which could be re-derived by setting the new parameter e (the charge of the central mass) to zero. Quite a lot of work has been done in explaining the Kerr-Newman metric as the metric of a charged black hole, but less effort has been made to reveal the structure of the theory. We will investigate some features of the exterior field of a charged rotating source.

In Sec. 2 we inspect the seed metric, the static auxiliary metric related to the Kerr-Newman metric. We describe how surfaces embedded in a 5-dimensional flat space, can be ascribed to this metric. Then we endow this metric with oblique-angled bein vectors to gain the rotating KN metric.

In Sec. 3 we investigate the KN metric by using a preferred reference system, the Carter system. We show that no magnetic fields are present for these observers and that the electric field has only one component in the radial direction. The gravitational field is coupled to a stress-energy tensor containing the electric field. We also examine both the gravitational field equations and the Maxwell equations. It turns out that the electric field has gravielectric interaction currents and sources.

In Sec. 4 we use two more preferred reference systems in relative motion with respect to the system treated in Sec. 3. In analogy to the theory of moving media in electrodynamics magnetic fields appear. It is shown that a [3+1]-splitting of the Einstein field equations leads to Maxwell-like equations for the gravitation. The field strengths have as currents and sources quadratic gravitational and electromagnetic terms. Furthermore, the electromagnetic fields are coupled to gravielectric and gravimagnetic interaction terms.

2. THE SEED METRIC

We start with the static seed metric, and we show that it is a metric of a double surface embedded in a 5-dimensional flat space. Five dimensions are sufficient for embedding but there is a need of six variables. We have faced the problem of embedding in several previous papers [3]. The metric reads as

$$ds^2 = \alpha_S^2 a_R^2 dr^2 + \Lambda^2 d\vartheta^2 + \sigma^2 d\varphi^2 + a_S^2 {dx^4}^2, \quad dx^4 = idt \;, \tag{2.1}$$

$$\begin{gathered} A^2 = r^2 + a^2, \quad \Lambda^2 = r^2 + a^2\cos^2\vartheta, \quad \delta^2 = r^2 + a^2 - 2Mr + e^2 \\ a_S = 1/\alpha_S = \frac{\delta}{A}, \quad a_R = 1/\alpha_R = \frac{\Lambda}{A} \end{gathered} \;. \tag{2.2}$$

M, a, and e are the mass parameter, the rotation parameter, and the charge. The metric reduces for e = 0 to the metric of the Kerr model, for a = 0 to the Reissner-Nordström metric, and for a = 0 and e = 0 to the Schwarzschild metric. We define by

$$a_S = \cos\varepsilon \tag{2.3}$$

the angle of ascent for a radial curve of a surface. For the sake of simplicity we take the orientation of ε to be cw. From

$$v_S^2 + a_S^2 = \sin^2\varepsilon + \cos^2\varepsilon = 1 \tag{2.4}$$

we get

$$v_S = \sin\varepsilon = -\sqrt{\frac{2M}{r} - \frac{e^2}{r^2}}\ , \tag{2.5}$$

the velocity of a freely falling observer. As the ascent of this radial curve is

$$\tan\varepsilon = -\frac{1}{\sqrt{\frac{r^2 + a^2}{2Mr - e^2} - 1}} \tag{2.6}$$

we set

$$dx_{hol}^{0'} = -\tan\varepsilon dr\ , \tag{2.7}$$

where $x^{0'}$ is the extra dimension in a 5-dimensional flat space. The solution of the integral

$$x^{0'}(r_1) = -\int_{r_0}^{r_1} \tan\varepsilon dr, \quad r_0 = M + \sqrt{M^2 - a^2 - e^2} \tag{2.8}$$

has no closed form, but can be integrated numerically. r_0 is he boundary of the geometry, called event horizon. The zero-plane of the surface is parameterized by the Boyer-Lindquist co-ordinates $\{r, \vartheta, \varphi\}$. The

$$x^{a'} = \left\{x^{0'}, r\cos\vartheta, A\sin\vartheta\cos\varphi, A\sin\vartheta\sin\varphi\right\} \tag{2.9}$$

are the rectilinear co-ordinates of a point on the surface. If we restrict ourselves to the first two dimensions of the expected metric we obtain a surface similar to the surface we found for the Kerr metric. Its parallels are ellipses and the image of the radial curves on the zero plane are hyperbolae. For a = 0 and e = 0 the surface reduces to Flamm's paraboloid. The parallels of the surface of the [r,φ]-slice are circles with radii $\sigma = A\sin\vartheta$.

The metric of this surface is not the KN metric. The reason is the following: Traveling on an elliptic parallel of the surface, the normal vector is commuting because the ascent of the elliptically squashed surface is position-dependent. Thus we endow the surface with a rigging vector, being normal to the surface only at the semi-minor axes of the elliptical parallels, where the surface is almost Schwarzschild-like. Transporting this vector around on a parallel, the vector does not change its angle of ascent. Evidently this vector is anholonomic. Its normal hyperplanes are not tangent to the surface. The family of these hyperplanes is part of the physical space. As

$$dx^{\underline{1}} = a_R dr$$

is the distance of two neighboring ellipses in a horizontal slice, we obtain

$$dx^{0'}_{anhol} = -\tan\varepsilon \; a_R(r,\vartheta)\, dr = -\tan\varepsilon \, dx^{\underline{1}} \;. \tag{2.10}$$

With (2.9) and the relation

$$ds^2 = dx^{0'^2} + dx^{1'^2} + dx^{2'^2} + dx^{3'^2}$$

we obtain

$$ds^2 = \left(\tan^2\varepsilon + 1\right) a_R^2 dr^2 + \Lambda^2 d\vartheta^2 + A^2 \sin^2\vartheta \, d\varphi^2 = \alpha_S^2 a_R^2 dr^2 + \Lambda^2 d\vartheta^2 + \sigma^2 d\varphi^2 \;,$$

the spacelike part of the seed metric. The curvature vector[1] of each 'radial' curve is

$$\rho_S = a_R A^3 \frac{\sqrt{2Mr - e^2}}{M\left(r^2 - a^2\right) - e^2 r} \;. \tag{2.11}$$

If we rewrite the timelike part of the seed metric to

$$a_S dx^4 = \rho_S \cos\varepsilon \, di\psi \tag{2.12}$$

we are able to explain the seed metric as the metric of a double surface embedded in a 5-dimensional flat space. More on this problem can be found in our previous papers [3].

[1] To be more precise, the only remaining component of the curvature vector in the reference system in use.

3. THE ROTATING METRIC

Rotating the third and fourth bein vectors on the physical surface, one obtains an oblique-angled 4-bein system. The metric describing the physical surface endowed with this new structure is the KN metric

$$ds^2 = {dx^1}^2 + {dx^2}^2 + \left[\alpha_R dx^3 + i\alpha_R \omega\sigma dx^4\right]^2 + a_S^2\left[-i\alpha_R \omega\sigma dx^3 + \alpha_R dx^4\right]^2 , \tag{3.1}$$

wherein

$$\begin{gathered} dx^1 = \alpha_S a_R dr, \quad dx^2 = \Lambda d\vartheta, \quad dx^3 = \sigma d\varphi, \quad dx^4 = \rho_S di\psi \\ \omega = \frac{a}{A^2}, \quad a_R^2 = 1 - \omega^2\sigma^2 \end{gathered} . \tag{3.2}$$

Firstly, we examine the metric with the Carter 4-bein (system C). Later on we will apply Lorentz transformations to set up the field equations for the system of Iyer and Kumar and also for the system of Bardeen. From the metric (3.1) we read the 4-bein vectors

$$\begin{matrix} \overset{1}{e}_1 = \alpha_S a_R, & \overset{2}{e}_2 = \Lambda, & \overset{3}{e}_3 = \alpha_R\sigma, & \overset{3}{e}_4 = i\alpha_R\omega\sigma, & \overset{4}{e}_3 = -ia_S\alpha_R\omega\sigma^2, & \overset{4}{e}_4 = a_S\alpha_R \\ \underset{1}{e}^1 = a_S\alpha_R, & \underset{2}{e}^2 = \dfrac{1}{\Lambda}, & \underset{3}{e}^3 = \dfrac{\alpha_R}{\sigma}, & \underset{3}{e}^4 = i\alpha_R\omega\sigma, & \underset{4}{e}^3 = -i\alpha_S\alpha_R\omega\,, & \underset{4}{e}^4 = \alpha_S\alpha_R \end{matrix} . \tag{3.3}$$

Evaluating the Ricci rotation coefficients we obtain the same structure we have already found for the Kerr geometry [3]. In this paper, we explicitly only show the new features. The gravitational force

$$G_m = \left\{\frac{1}{\rho_S}\tan\varepsilon, 0, 0, 0\right\} \tag{3.4}$$

has two contributions

$$G_m = E_m + \mathcal{E}_m \;, \tag{3.5}$$

wherein

$$E_1 = -\alpha_S\alpha_R \frac{M}{A^4}\left(r^2 - a^2\right) \tag{3.6}$$

is the Kerr-like attractive gravitational force and

$$\mathcal{E}_1 = \alpha_S \alpha_R \frac{r}{A}\frac{e^2}{A^3} \tag{3.7}$$

is repulsive and due to the charge of the source. From the rotational part of the metric we derive the centrifugal force

$$F_m = \alpha_R^2 \omega^2 \sigma \sigma_{|m} \tag{3.8}$$

and the rotational force

$$\begin{gathered} \Omega^C_{nm} = -\left[H^C_{mn} + D^C_{mn}\right], \quad H^C_{mn} = 2\left[ia_S \alpha_R^2 \omega \sigma_{|[m} c_{n]} + ia_S \alpha_R^2 \sigma \omega_{|[m} c_{n]}\right] \\ D^C_{mn} = 2i\alpha_S \alpha_R^2 \sigma \omega_{|(m} c_{n)}, \quad c_n = \{0,0,1,0\} \end{gathered} . \tag{3.9}$$

The terms with $\omega_{|1}\sigma = -2\omega\sigma_{|1}, \ \omega_{|2}\sigma = 0$ are the contributions of the differential rotation law. The symmetric one is responsible for the shears of the observer fields. The curvature quantities

$$B_m = \left\{\frac{a_S}{\rho_E},0,0,0\right\}, \quad N_m = \left\{0,\frac{1}{\rho_H},0,0\right\}, \quad C_m = \left\{\frac{1}{\sigma}\sigma_{|1},\frac{1}{\sigma}\sigma_{|2},0,0\right\} \tag{3.10}$$

are obtained from the spacelike part of the metric. In (3.10) the

$$\rho_E = \frac{\Lambda^3}{Ar}, \quad \rho_H = -\frac{\Lambda^3}{a^2 \sin\vartheta \cos\vartheta}, \quad \sigma = A\sin\vartheta \tag{3.11}$$

are the nonvanishing components of the elliptical, hyperbolical, and circular curvature vectors of the surface. With the curvature quantities the Einstein equations

$$R_{mn} - \frac{1}{2} R g_{mn} = -\kappa T_{mn}$$

are satisfied if

$$\kappa T_{mn} = \begin{pmatrix} \mathcal{F}_C^2 & & & \\ & -\mathcal{F}_C^2 & & \\ & & -\mathcal{F}_C^2 & \\ & & & \mathcal{F}_C^2 \end{pmatrix}, \tag{3.12}$$

wherein

$$\mathcal{F}^C_m = \left\{\frac{e}{\Lambda^2},0,0,0\right\} \tag{3.13}$$

is the electric field strength[2]. Rescaling the electromagnetic field with

$$e^2 \to \frac{\kappa}{2} e^2$$

the stress-energy tensor can be written as

$$T_{mn} = \mathcal{F}_m{}^s \mathcal{F}_{ns} - \frac{1}{4} g_{mn} \mathcal{F}^{rs} \mathcal{F}_{rs}, \quad \mathcal{F}_{14} = \mathcal{F}^{C}_{1} . \tag{3.14}$$

The Einstein field equations can be calculated as we have done in our papers [3]. We only note the [33], [34], and [44]-components[3]

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[2] Misner, Thorne and Wheeler [4] write for the electromagnetic field tensor

$$\mathbf{F} = e\Lambda^{-4}\left(r^2 - a^2\cos^2\vartheta\right)\mathbf{d}r \wedge \left[\mathbf{d}t - a^2\sin^2\vartheta\,\mathbf{d}\varphi\right] + 2e\Lambda^{-4} ar\cos\vartheta\sin\vartheta\,\mathbf{d}\vartheta \wedge \left[\left(r^2 + a^2\right)\mathbf{d}\varphi - a\mathbf{d}t\right].$$

Evidently this expression contains the components of the electromagnetic field strengths with respect to the oblique-angled co-ordinate system of the Kerr geometry:

$$\begin{aligned} \mathcal{F}_{13} &= -\mathcal{F}^{C}_{1}\omega\sigma^2 \frac{r^2 - a^2\cos^2\vartheta}{\Lambda^2}, \quad & \mathcal{F}_{14} &= \mathcal{F}^{C}_{1}\frac{r^2 - a^2\cos^2\vartheta}{\Lambda^2} \\ \mathcal{F}_{23} &= \mathcal{F}^{C}_{1}\frac{2ar\cos\vartheta\, A^2\sin\vartheta}{\Lambda^2}, \quad & \mathcal{F}_{24} &= -\mathcal{F}^{C}_{1}\frac{2ar\sin\vartheta\, a\sin\vartheta}{\Lambda^2} \end{aligned} .$$

Re-writing these quantities in an orthogonal reference system, say the Carter system, one obtains

$$\begin{aligned} \mathcal{F}_{13} &= 0, \quad & \mathcal{F}_{14} &= \mathcal{F}^{C}_{1}\frac{r^2 - a^2\cos^2\vartheta}{\Lambda^2} \\ \mathcal{F}_{23} &= \mathcal{F}^{C}_{1}\frac{2ra\cos\vartheta}{\Lambda^2}, \quad & \mathcal{F}_{24} &= 0 \end{aligned} .$$

The remaining two components, the magnetic field $\mathcal{F}_{23}$ and the electric field $\mathcal{F}_{14}$ are parallel and point into the radial direction. They have a magnetic and an electric monopole as source. It seems that the Newman-Janis algorithm has not been properly translated into ordinary tensor formalism. Since

$$\Lambda^2 = r^2 + a^2\cos^2\vartheta = \left(r + ia\cos\vartheta\right)\left(r - ia\cos\vartheta\right),$$

we combine the two components $\mathcal{F}_{23}$ and $\mathcal{F}_{14}$ to

$$\left(\mathcal{F}_{14}\mathcal{F}_{14} - \mathcal{F}_{23}\mathcal{F}_{23}\right) = \left(\mathcal{F}_{14} + \mathcal{F}_{23}\right)\left(\mathcal{F}_{14} - \mathcal{F}_{23}\right) = \mathcal{F}^{C}_{1}\mathcal{F}^{C}_{1} .$$

Only one electric component $\mathcal{F}^{C}_{1}$ remains, pointing into the radial direction.

[3] $||_4$ is the fourth graded derivative [3] and corresponds to the spacelike covariant derivative.

$$C^{s}{}_{||s} + F^{s}{}_{||s} = \Omega^{rs}_{C}\Omega^{C}_{sr} - \mathcal{F}^{s}_{\ C}\mathcal{F}^{C}_{\ s}$$

$$\Omega^{ns}_{C\,\underset{4}{||}n} = 2\,\Omega^{[sn]}_{C}F_{s} \quad . \tag{3.15}$$

$$G^{s}{}_{||s} + F^{s}{}_{||s} = \Omega^{rs}_{C}\Omega^{C}_{sr} - \mathcal{F}^{s}_{\ C}\mathcal{F}^{C}_{\ s}$$

On the right side in these equations are the total field stress, the field current, and the field energy. In the system of Carter (system C) no magnetic field is present. Evidently, the observers of this system are differentially co-rotating with the electric sources.

From the covariant definition of the electric field tensor

$$\mathcal{F}_{mn} = 2\mathcal{F}^{C}_{\ [m}u_{n]} \tag{3.16}$$

and the covariant definition of the time derivative

$$\frac{d}{id\tau}\mathcal{F}^{C}_{\ m} = \mathcal{F}^{C}_{\ m||n}u^{n} \tag{3.17}$$

we obtain

$$\mathcal{F}^{\ \alpha}_{C\underset{4}{||}\alpha} = 0, \quad \frac{d}{id\tau}\mathcal{F}^{C}_{\ \alpha} = D^{C}_{\alpha\beta}\mathcal{F}^{\beta}_{\ C}, \quad \alpha,\beta = 1,2,3 \ . \tag{3.18}$$

The second set can be written as

$$\mathcal{F}_{<mn||r>} = 2\,\Omega^{C}_{<mn}\mathcal{F}^{C}_{\ r>} \ . \tag{3.19}$$

or

$$\mathcal{F}_{<\alpha\beta\underset{4}{||}\gamma>} \equiv 0, \quad \mathcal{F}^{C}_{\ [\alpha||\beta]} = \mathcal{F}^{C}_{\ [\alpha}F_{\beta]} \ . \tag{3.20}$$

While the first relation is trivial, the second relation

$$\mathrm{rot}\,\vec{\mathcal{F}} = \vec{\mathcal{F}}\times\vec{F}$$

shows that the electric field is coupled to a gravimagnetic interaction current. With these equations we are able to verify

$$T_{m\ \ ||n}^{\ \ n} = \mathcal{F}_{m}^{\ \ s}\mathcal{F}^{n}_{\ \ s||n} - \frac{1}{2}\mathcal{F}^{ns}\mathcal{F}_{<mn||s>} = 0 \ . \tag{3.21}$$

Choosing other preferred reference systems, relatively rotating with respect to the Carter system, we expect magnetic field contributions in analogy to the theory of moving media in electrodynamics. We will investigate this case in the next Section.

4. OBSERVING MAGNETIC FIELDS

Despite the Carter system, two more preferred reference systems are utilized for the Kerr metric. They can be applied to the KN metric as well. Defining the angular velocities

$$\omega_{AC} = \alpha_S \omega , \quad \omega_{BC} = a_S \omega \ , \tag{4.1}$$

the circular velocities, and the Lorentz factors

$$\alpha_{AC}\omega_{AC}\sigma, \quad \alpha_{BC}\omega_{BC}\sigma, \quad \alpha_{AC} = 1\Big/\sqrt{1-\omega^2_{AC}\sigma^2}\,, \quad \alpha_{BC} = 1\Big/\sqrt{1-\omega^2_{BC}\sigma^2} \tag{4.2}$$

one is able to introduce new observer fields used by Iyer and Kumar [5] and Bardeen [6], respectively. We will call them system A and system B. The observers of system A have the properties

$$u_{(\alpha\,||\beta)} = 0 \ . \tag{4.3}$$

Their motion is free of shears and expansion. The system B is the locally non-rotating system (LNR) with

$$u_{[\alpha\,||\beta]} = 0 \ . \tag{4.4}$$

These observers experience shears having their origin in the differential rotation law, but they don't experience Coriolis-like forces. Applying the Lorentz transformation

$$L_3^{3'} = \alpha_X , \quad L_3^{4'} = i\alpha_X\omega_X\sigma, \quad L_4^{3'} = -i\alpha_X\omega_X\sigma, \quad L_4^{4'} = \alpha_X , \quad \text{X = AC or X = BC} \tag{4.5}$$

to the electric field one obtains

$$\mathcal{H}^A_{13} = -i\alpha_{AC}\omega_{AC}\sigma\mathcal{F}^C_1, \quad \mathcal{F}^A_1 = \alpha_{AC}\mathcal{F}^C_1, \quad \mathcal{H}^B_{13} = -i\alpha_{BC}\omega_{BC}\sigma\mathcal{F}^C_1, \quad \mathcal{F}^B_1 = \alpha_{BC}\mathcal{F}^C_1 \ . \tag{4.6}$$

These quantities are components[4] of

$$\mathcal{F}_{mn} = \mathcal{H}_{mn} + 2\,\mathcal{F}_{[m}u_{n]} \ . \tag{4.7}$$

The magnetic field can be written as

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[4] In the following the markers A and B are suppressed.

$$\vec{\mathcal{H}} = \alpha\left[\vec{v}\times\vec{\mathcal{F}}\right], \quad \vec{v} = \vec{\omega}\times\vec{\sigma} \tag{4.8}$$

for both systems. After rescaling, the components of the stress-energy tensor are

$$T_{11} = -T_{22} = \frac{1}{2}\left[\mathcal{F}_1\mathcal{F}_1 + \mathcal{H}_{13}\mathcal{H}_{13}\right], \quad T_{44} = -T_{33} = \frac{1}{2}\left[\mathcal{F}_1\mathcal{F}_1 - \mathcal{H}_{13}\mathcal{H}_{13}\right], \quad T_{34} = \mathcal{F}_1\mathcal{H}_{13} \ . \tag{4.9}$$

The Maxwell equations for the system A decompose into

$$\begin{aligned} &\mathcal{F}^{m}_{A\underset{4}{\|}m} = \Omega^{mn}_{AC}\mathcal{H}^{A}_{mn}, \quad \mathcal{H}^{m}_{An\underset{4}{\|}m} = -\left[\Omega^{AC}_{mn} + \Omega_{mn}\right]\mathcal{F}^{m}_{A} \\ &\mathcal{H}^{A}_{<\alpha\beta\underset{4}{\|}\gamma>} = 2\mathcal{F}^{A}_{<\alpha}\Omega^{AC}_{\beta\gamma>}, \quad \mathcal{F}^{A}_{[\alpha\underset{4}{\|}\beta]} = \left[\Omega^{AC}_{[\alpha} + F_{[\alpha}\right]\mathcal{F}^{A}_{\beta]} \end{aligned} \ . \tag{4.10}$$

The electric and magnetic fields are coupled to gravielectric interaction currents and sources. The gravitational part consists of Coriolis-like terms, centrifugal terms, and others due to the differential rotation law

$$\begin{aligned} \Omega^{AC}_{mn} &= 2i\alpha^2_{AC}\omega_{AC}\sigma_{|[m}c_{n]} + 2i\alpha^2_{AC}\omega_{AC|[m}c_{n]}\sigma \\ \Omega^{AC}_{m} &= \alpha^2_{AC}\omega^2_{AC}\sigma\sigma_{|m} + \alpha^2_{AC}\omega_{AC}\omega_{AC|m}\sigma^2 \end{aligned} \tag{4.11}$$

and the skew-symmetric quantity derived from the system C

$$\Omega_{3m} = -\Omega_{m3} = -\Omega^{C}_{3m} \ . \tag{4.12}$$

For the system B one obtains

$$\begin{aligned} &\mathcal{F}^{m}_{B\underset{4}{\|}m} = D_m\mathcal{F}^{m}_{B}, \quad \mathcal{H}^{m}_{Bn\underset{4}{\|}m} = -2\Omega^{BC}_{mn}\mathcal{F}^{m}_{B} + G^{B}_{m}\mathcal{H}^{m}_{Bn} \\ &\mathcal{H}^{B}_{<\alpha\beta\underset{4}{\|}\gamma>} = 2\mathcal{F}^{B}_{<\alpha}H^{C}_{\beta\gamma>}, \quad \mathcal{F}^{B}_{[\alpha\underset{4}{\|}\beta]} = \mathcal{F}^{B}_{[\alpha}\left[\Omega^{BC}_{\beta]} + F_{\beta]}\right] \end{aligned} \ . \tag{4.13}$$

The quantity H^{C}_{mn} is defined by Eq. (3.9) and

$$\begin{aligned} \Omega^{BC}_{m3} &= i\alpha^2_{BC}\omega_{BC}\sigma_{|m} + i\alpha^2_{BC}\omega_{BC|m}\sigma \\ \Omega^{BC}_{m} &= \alpha^2_{BC}\omega^2_{BC}\sigma\sigma_{|m} + \alpha^2_{BC}\omega_{BC}\omega_{BC|m}\sigma^2 \end{aligned} \ . \tag{4.14}$$

The radial force

$$D_m = \alpha^2_R\omega\,\omega_{|m}\sigma^2 \tag{4.15}$$

has its origin in the differential rotation law and

$$G^{B}_{m} = G_m - \Omega^{BC}_{m} + F_m + D_m \tag{4.16}$$

is a combination of all radial forces. The quantities Ω^{BC}_{mn} and Ω^{BC}_{m} have the same structure as (4.11). For the Maxwell equations in both systems one can write

$$\operatorname{div}\vec{\mathcal{F}} = \rho_{eg}, \quad \operatorname{rot}\vec{\mathcal{F}} = \vec{j}_{mg}, \quad \operatorname{div}\vec{\mathcal{H}} = \rho_{mg}, \quad \operatorname{rot}\vec{\mathcal{H}} = \vec{j}_{eg} \ . \tag{4.17}$$

On the right side of these equations are the gravielectric and gravimagnetic currents and sources.

The Einstein field equations can be treated in the same manner as we have done in pervious papers [3]. We only note some results.

$$G^{A}_{m} = G_m + \Omega^{AC}_{m} - \left(F_m + D_m\right) \tag{4.18}$$

is the collection of all radial forces and

$$\Omega^{A}_{mn} = \Omega^{AC}_{mn} - \Omega_{mn} \tag{4.19}$$

is the rotational force for the system A. From the Einstein field equations one obtains

$$\begin{aligned} &\Omega^{s}_{A n \underset{4}{\|} s} = j_n, \quad j_n = 2\left[\Omega^{s}_{An} G^{A}_{s} - \mathcal{H}^{s}_{An} \mathcal{F}^{A}_{s}\right] \\ &G^{m}_{A \underset{4}{\|} m} = j, \quad j = \left[G^{m}_{A} G^{A}_{m} - \Omega^{A}_{mn} \Omega^{mn}_{A}\right] - \left[\mathcal{F}^{A}_{m} \mathcal{F}^{m}_{A} - \mathcal{H}^{A}_{m3} \mathcal{H}^{m3}_{A}\right] \end{aligned} \ . \tag{4.20}$$

The total field current and the total field energy, both having gravitational and electromagnetic contributions, are conserved

$$j^{n}{}_{\underset{4}{\|} n} = 0, \quad j^{\cdot} = 0 \ . \tag{4.21}$$

For the system B one obtains the equations

$$\begin{aligned} &D^{s}_{B n \underset{4}{\|} s} = j_n, \quad j_n = -2\mathcal{H}^{s}_{Bn} \mathcal{F}^{B}_{s} \\ &G^{m}_{B \underset{4}{\|} m} = j, \quad j = \left[G^{m}_{B} G^{B}_{m} + D^{B}_{mn} D^{mn}_{B}\right] - \left[\mathcal{F}^{B}_{m} \mathcal{F}^{m}_{B} - \mathcal{H}^{B}_{m3} \mathcal{H}^{m3}_{B}\right] \end{aligned} \ . \tag{4.22}$$

In these equations the tensorial field strength appears only in the symmetric form

$$D^{B}_{\alpha\beta} = -A_{\beta\alpha}{}^{4}, \quad A_{[\beta\alpha]}{}^{4} = 0, \qquad \alpha,\beta = 1,2,3 \ . \tag{4.23}$$

Since

$$u_{[\alpha||\beta]} = 0, \quad u_{(\alpha||\beta)} = D^{B}_{\alpha\beta} \tag{4.24}$$

the observer B experiences no Coriolis-like forces, but shears $D^{B}_{\alpha\beta}$. The current does not contain any gravitational self-interaction terms. The charge and the current are conserved.

5. CONCLUSIONS

It is of some interest to treat the gravitational field equations in the same manner as we do in classical mechanics or electrodynamics. The field equations of the Kerr-Newman metric, decomposed in spacelike and timelike parts, exhibit currents and sources quadratic in the gravitational and electromagnetic field strengths. The Maxwell equations show that the field strengths are coupled to gravielectric interaction terms. We have calculated all these equations for reference systems in relative rotation.

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