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GEOMETRICAL INTERPRETATION OF THE NUT METRIC

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Contents

We re-write the NUT metric, perform a [3+1]-decomposition of the field equations, and we embed it into a 5-dimensional flat space. Thus, we prepare the model for adding an interior.

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1. INTRODUCTION

Newman, Unti, and Taburino [1] found a new gravitational model in 1963. It is a vacuum solution in close relation to the Schwarzschild model and has an additional parameter l, being interpreted as monopole parameter. We reformulate the NUT theory in such a way that the field equations appear in an invariant tensorial form and we embed the metric into a 5-dimensional flat space. Thus, we are able to simplify the field equations. Moreover, we can derive an interior for the NUT metric by use of this method. We will show this in a subsequent paper.

The field equations contain a rotational two-rank tensor, which has only one component. The dual vector of this quantity points into the radial direction. It has properties of a magnetic monopole but has to be interpreted as gravitational action. The model has no counterpart in nature, but is of some interest as it can be assigned to the Kerr family. This model is also related to the Reissner-Nordström solution and can be combined with the Kerr model [2].

We analyze the NUT metric in several steps. Firstly, we gain, by an intrinsic transformation, the seed metric, which is free of rotational effects. This metric is simplified once more. This simplified metric is in close relation to the Reissner-Nordström metric. It will be interpreted as a metric of a surface embedded into a 5-dimensional flat space.

In Sec. 2 we investigate the reduced metric. In Sec. 3 we embed this metric into a 5-dimensional flat space. In Sec. 4 we analyze the seed metric and present the field equations of the NUT metric in a tensorial form. We perform a [3+1]-decomposition of these equations.

2. THE REDUCED METRIC

The NUT metric has the form

$$ds^2 = \frac{A^2}{\delta^2}dr^2 + A^2 d\vartheta^2 + A^2\sin^2\vartheta d\varphi^2 - \frac{\delta^2}{A^2}\left[dt + 2l(1-\cos\vartheta)d\varphi\right]^2 . \qquad (2.1)$$

$$A^2 = r^2 + l^2 \qquad \delta^2 = r^2 - 2Mr - l^2$$

Putting the monopole parameter l zero one obtains the Schwarzschild metric. It is easy to remove the second term in the bracket of (2.1) by an intrinsic transformation and to restore it later by the same transformation. Doing so we obtain the static seed metric

$$ds^2 = \frac{A^2}{\delta^2}dr^2 + A^2 d\vartheta^2 + A^2\sin^2\vartheta d\varphi^2 + \frac{\delta^2}{A^2}{dx^4}^2, \quad dx^4 = idt\ . \qquad (2.2)$$

Evidently, the radial variable r does not fit the expected radii of the great circles of the spherically symmetric system. Therefore, we split the metric factors in (2.2)

$$\alpha_G = \frac{r}{\delta}, \quad a_G = \frac{\delta}{r}, \quad \alpha_D = \frac{A}{r}, \quad a_D = \frac{r}{A} \tag{2.3}$$

and we re-write the metric in a more transparent form

$$ds^2 = \alpha_D^2\left[\alpha_G^2 dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2\right] + a_D^2 a_G^2 dit^2 \ . \tag{2.4}$$

α_G is the gravitational factor related to the corresponding factor of the Schwarzschild theory. α_D deforms the metric in dependence on the distance to the center of gravitation. If one translates Weyl's geometry into tetrad formalism one finds similarities to the NUT model. In Weyl's geometry, the quantities are measured with space-time dependent rods and clocks. This dependency exceeds the concept of Riemannian geometry and determines an additional set of equations which have Maxwellian structure. Weyl has found a unified theory of gravitation and electromagnetism which was not successful.

The reduced metric

$$ds^2 = \alpha_G^2 dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 + a_G^2 dit^2 \tag{2.5}$$

differs from the Reissner-Nordström metric only by the content of the factor α_G. With the substitution $l \to ie$, one obtains from (2.5) the Reissner-Nordström metric. From (2.5) we read the components of the 4-bein vectors

$$\overset{1}{e}_1 = \alpha_G, \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = r\sin\vartheta, \quad \overset{4}{e}_4 = a_G \tag{2.6}$$

and the partial derivatives

$$\partial_1 = a_G\frac{\partial}{\partial r}, \quad \partial_2 = \frac{\partial}{r\partial\vartheta}, \quad \partial_3 = \frac{\partial}{r\sin\vartheta\partial\varphi}, \quad \partial_4 = \frac{\partial}{a_G\partial it} \ . \tag{2.7}$$

The Ricci-rotation coefficients split into

$$\begin{gathered} A_{mn}{}^s = B_{mn}{}^s + C_{mn}{}^s + E_{mn}{}^s \\ B_{mn}{}^s = b_m B_n b^s - b_m b_n B^s, \quad C_{mn}{}^s = c_m C_n c^s - c_m c_n C^s, \quad E_{mn}{}^s = -\left[u_m E_n u^s - u_m u_n E^s\right] \ , \end{gathered} \tag{2.8}$$

wherein

$$b_m = \{0,1,0,0\}, \quad c_m = \{0,0,1,0\}, \quad u_m = \{0,0,0,1\} \tag{2.9}$$

are unit vectors.

$$B_1 = \frac{a_G}{r}, \quad C_1 = \frac{a_G}{r}, \quad C_2 = \frac{1}{r}\cot\vartheta, \quad E_1 = -\alpha_G\frac{M}{r^2} - \alpha_G\frac{l^2}{r^3} \tag{2.10}$$

are the components of the field strengths describing the curvatures of the space. The gravitational force E has only one component in the radial direction and consists of the Schwarzschild-like part and an additional attractive term, quadratic in the NUT parameter and decreasing in r with the third order. With the identification

$$a_G = \cos\varepsilon \tag{2.11}$$

one obtains the angle of ascent of the radial curves of a surface which is similar to Flamm's paraboloid of the Schwarzschild geometry. This surface will be embedded into a 5-dimensional flat space. With the last relation and

$$v_G^2 = 1 - a_G^2 \tag{2.12}$$

one obtains also the velocity of a freely falling observer[1]

$$v_G = \sin\varepsilon = -\sqrt{\frac{2M}{r} + \frac{l^2}{r^2}} \tag{2.13}$$

and with

$$\tan\varepsilon = \frac{v_G}{a_G} = -\sqrt{\frac{2Mr + l^2}{r^2 - 2Mr - l^2}} \tag{2.14}$$

the curvature radius of the radial lines of the above-mentioned surface

$$\rho = \frac{\sqrt{2Mr + l^2}}{Mr + l^2} r^2 = -v_G \frac{r^3}{Mr + l^2} \ . \tag{2.15}$$

We note the useful relations

$$v_{G|1} = \frac{a_G}{\rho}, \quad a_{G|1} = -\frac{v_G}{\rho} \ . \tag{2.16}$$

With (2.14) and (2.15) we are able to re-write the gravitational field strength as

$$E_1 = \frac{1}{\rho}\tan\varepsilon \ . \tag{2.17}$$

Facing the 5-dimensional formulation of the theory we intuitively add the extra components of the quantities defined in (2.10) and (2.17)

$$M_0 = \frac{1}{\rho}, \quad B_0 = \frac{v_G}{r}, \quad C_0 = \frac{v_G}{r}, \quad E_0 = -\frac{1}{\rho} \ . \tag{2.18}$$

With the help of the unit vector of the radial direction

$$m_m = \{1,0,0,0\} \tag{2.19}$$

and the previously defined unit vectors (2.9) we obtain for the Ricci of the reduced metric

[1] ε is oriented cw.

$$
\begin{aligned}
R_{mn} = & -\left[B_{n\underset{2}{||}m} + B_n B_m \right] - b_n b_m \left[B^{s}{}_{\underset{2}{||}s} + B^s B_s \right] \\
& -\left[C_{n\underset{3}{||}m} + C_n C_m \right] - c_n c_m \left[C^{s}{}_{\underset{3}{||}s} + C^s C_s \right] \\
& +\left[E_{n\underset{4}{||}m} - E_n E_m \right] + u_n u_m \left[E^{s}{}_{\underset{4}{||}s} - E^s E_s \right] \\
= & \, m_m m_n \left(M_0 B_0 + M_0 C_0 - M_0 E_0 \right) \\
& + b_m b_n \left(B_0 M_0 + B_0 C_0 - B_0 E_0 \right) \\
& + c_m c_n \left(C_0 M_0 + C_0 B_0 - C_0 E_0 \right) \\
& - u_m u_n \left(E_0 M_0 + E_0 B_0 + E_0 C_0 \right) \\
& - \left(m_m m_n + u_m u_n \right) E_1 \frac{1}{\rho} \rho_{|1}
\end{aligned} \quad . \tag{2.20}
$$

The graded derivatives therein are defined by

$$
\begin{aligned}
& m_{m\underset{1}{||}n} = m_{m|n} = 0, \quad b_{m\underset{2}{||}n} = b_{m|n} = 0 \\
& c_{m\underset{3}{||}n} = c_{m|n} - B_{nm}{}^{s} c_s = 0, \quad u_{m\underset{4}{||}n} = u_{m|n} - B_{nm}{}^{s} u_s - C_{nm}{}^{s} u_s = 0
\end{aligned} \quad . \tag{2.21}
$$

A more detailed substantiation of these definitions one can find in our papers collected in [3].

The second block in the Ricci (2.20) mainly consists of the second fundamental forms of the theory of surfaces. Calculating with (2.15) and (2.18) the auxiliary relations

$$
B_0 E_0 = C_0 E_0 = \frac{Mr + l^2}{r^4}, \quad B_0 C_0 = \frac{2Mr + l^2}{r^4}, \quad M_0 = -E_0
$$

we obtain

$$
B_0 M_0 + B_0 C_0 - B_0 E_0 = C_0 M_0 + C_0 B_0 - C_0 E_0 = -\tilde{\omega}^2
$$

with the monopole field strength

$$
\tilde{\omega} = \frac{l}{r^2} \quad . \tag{2.22}
$$

The computation of the last term in (2.20) is more complex. With (2.15) one obtains

$$
\begin{aligned}
E_1 \frac{1}{\rho} \rho_{|1} = & \quad M_0 B_0 + M_0 C_0 - M_0 E_0 - \tilde{\omega}^2 \\
= & -\left[E_0 M_0 + E_0 B_0 + E_0 C_0 \right] - \tilde{\omega}^2
\end{aligned} \quad . \tag{2.23}
$$

Inserting the preceding relations we can write the Ricci of the reduced metric as

$$R_{mn} = \begin{pmatrix} \tilde{\omega}^2 & & & \\ & -\tilde{\omega}^2 & & \\ & & -\tilde{\omega}^2 & \\ & & & \tilde{\omega}^2 \end{pmatrix} . \tag{2.24}$$

It follows immediately $R = 0$. Rescaling the NUT parameter with $l^2 \to \frac{\kappa}{2} l^2$ and defining the gravimagnetic field strength by

$$\mathcal{H}_{23} = -i\tilde{\omega} \tag{2.25}$$

one obtains the dual vector of this quantity

$$\mathcal{H}_\alpha = i\varepsilon_\alpha{}^{\beta\gamma}\mathcal{H}_{\beta\gamma}, \quad \mathcal{H}_\alpha = \{\tilde{\omega}, 0, 0\}, \quad \alpha = 1, 2, 3 \ . \tag{2.26}$$

By analogy with the electrodynamics, one can construct the energy-momentum tensor

$$T_{mn} = -\left[\mathcal{H}_m{}^s\mathcal{H}_{ns} - \frac{1}{4} g_{mn}\mathcal{H}^{rs}\mathcal{H}_{rs}\right] \tag{2.27}$$

in accordance with the Ricci (2.24). The quantity $\mathcal{H}$ satisfies the Maxwellian equations

$$\mathcal{H}^s{}_{n||s} = 0. \quad \mathcal{H}_{<mn||s>} = 0 \ . \tag{2.28}$$

The reduced NUT metric may be combined with the Reissner-Nordström metric. Calculating this model is simple as it differs only a little from the reduced NUT model. For the Ricci one gets the components

$$R_{11} = -R_{22} = -R_{33} = R_{44} = \frac{l^2}{r^4} - \frac{e^2}{r^4} \ . \tag{2.29}$$

Because of the opposite signs of the NUT and the charged term, one is not able to compare the combined stress-energy tensor with the one of the electrodynamics, composed of electric and magnetic parts. The l- and e-terms belong to different worlds. Let us treat the gravitational force (2.17) once more. With (2.14) and (2.15) we assess that it consists of two parts

$$E_1 = -\alpha_G \frac{M}{r^2} - \alpha_G \frac{l^2}{r^3} \tag{2.30}$$

both being attractive. The first part corresponds to the Schwarzschild force of gravity and coincides with it for l = 0. The second part follows the law $1/r^3$ and has a shorter range. For this quantity, the monopole parameter is substantial. If one defines

$$\mathcal{E}_1 = -\alpha_G \frac{l^2}{r^3} \tag{2.31}$$

a short calculation[2]

$$\mathcal{C}^{n}{}_{||n} = \tilde{\omega}^2 \tag{2.32}$$

shows that the quantity $\mathcal{C}$ is coupled to a monopole source. For the Schwarzschild-like force one has

$$\bar{E}_1 = -\alpha_G \frac{M}{r^2}, \quad \bar{E}^{n}{}_{||n} = 0 \tag{2.33}$$

in accordance with the fourth line of (2.24).

3. FIVE-DIMENSIONAL REPRESENTATION OF THE REDUCED METRIC

Having treated the 4-dimensional field equations of the reduced NUT metric, we are able to write down the 5-dimensional field equations by rearranging the field equations and supplementing the field strengths with extra components. In view of technical reasons, we already have introduced the 0-components of the field strengths (2.18). The 0-direction is the *local* extra dimension in a 5-dimensional[3] flat space. We have to bear in mind that one has to use alternatively two operators [3]

$$\partial_0 = \frac{\partial}{\partial \rho}, \quad \partial_0 = v_G \frac{\partial}{\partial r} \tag{3.1}$$

for the directional derivative in the extra dimension, depending on the kind of quantity being differentiated. The 5-dimensional Ricci (a=0,1,…,4) reads as

$$\begin{aligned} {}^5R_{ab} = &-\left[M_{\underset{1}{b|||a}} - M_{\underset{1}{b|||c}} m^c m_a + M_b M_a\right] - m_a m_b \left[M^{c}{}_{\underset{1}{|||c}} + M^c M_c\right] \\ &-\left[B_{\underset{2}{b|||a}} + B_b B_a\right] - b_a b_b \left[B^{c}{}_{\underset{2}{|||c}} + B^c B_c\right] \\ &-\left[C_{\underset{3}{b|||a}} + C_b C_a\right] - c_a c_b \left[C^{c}{}_{\underset{3}{|||c}} + C^c C_c\right] \\ &+\left[E_{\underset{4}{b|||a}} - E_b E_a\right] + u_a u_b \left[E^{c}{}_{\underset{4}{|||c}} - E^c E_c\right] \end{aligned} \tag{3.2}$$

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[2] The relation $B_1 \mathcal{C}_1 = \tilde{\omega}^2$ is useful.

[3] In papers [3], we have shown that 5-dimensional embeddings for the whole Kerr family are possible and fruitful. The use of a *double-surface* theory does not violate the theorems of Kasner and Eisenhart.

and satisfies the relation

$$ {}^{5}R_{ab} + 2\,X_{fb}{}^{d}\wp^{\,f}_{[d|||a]} = 0 \ . \tag{3.3} $$

The $X_{fb}{}^{d}$ are the connexion coefficients of a pseudo hypersphere [3] and the $\wp$ are the projectors[4]

$$ \wp^{0}_{0} = \wp^{1}_{1} = \wp^{4}_{4} = \frac{X}{\rho}, \quad \wp^{2}_{2} = \wp^{3}_{3} = \frac{X v_G}{r} \ . \tag{3.4} $$

The graded derivatives in (3.2) are defined as

$$ \begin{gathered} m_{a\underset{1}{|||}b} = m_{a|b} = 0, \quad b_{a\underset{2}{|||}b} = b_{a|b} - M_{ba}{}^{c} b_c = 0 \\ c_{a\underset{3}{|||}b} = c_{a|b} - M_{ba}{}^{c} c_c - B_{ba}{}^{c} c_c = 0, \quad u_{a\underset{4}{|||}b} = u_{a|b} - M_{ba}{}^{c} u_c - B_{ba}{}^{c} u_c - C_{ba}{}^{c} u_c = 0 \end{gathered} \ . \tag{3.5} $$

The 5-dimensional connexion coefficients split into

$$ A_{ab}{}^{c} = M_{ab}{}^{c} + B_{ab}{}^{c} + C_{ab}{}^{c} + E_{ab}{}^{c} \tag{3.6} $$

with the new quantity

$$ M_{ab}{}^{c} = m_a M_b m^c - m_a m_b M^c, \quad M_a = \left\{ \frac{1}{\rho}, 0, 0, 0, 0 \right\} \tag{3.7} $$

referring to the extrinsic geometry of the model. All brackets in (3.2) vanish except the two last ones. They have to be calculated with

$$ E_{b\underset{4}{|||}a} - E_b E_a = -E_b \frac{1}{\rho} \rho_{|\underline{a}}, \quad E^{c}{}_{\underset{4}{|||}c} - E^{c} E_c = -E^{c} \frac{1}{\rho} \rho_{|\underline{c}}, \quad \underline{a} = 1,2,3,4 \ . \tag{3.8} $$

Resolving (3.3) with (3.4) one obtains

$$ \begin{aligned} {}^{5}R_{11} + 2X_{f1}{}^{d}\wp^{\,f}_{[d|||1]} &= {}^{4}R_{11} - \left[M_0 B_0 + M_0 C_0 - M_0 E_0 \right] + E_1 \frac{1}{\rho} \rho_{|1} = {}^{4}R_{11} - \tilde{\omega}^2 = 0 \\ {}^{5}R_{22} + 2X_{f2}{}^{d}\wp^{\,f}_{[d|||2]} &= {}^{4}R_{22} - \left[B_0 M_0 + B_0 C_0 - B_0 E_0 \right] = {}^{4}R_{22} + \tilde{\omega}^2 = 0 \\ {}^{5}R_{33} + 2X_{f3}{}^{d}\wp^{\,f}_{[d|||3]} &= {}^{4}R_{33} - \left[C_0 M_0 + C_0 B_0 - C_0 E_0 \right] = {}^{4}R_{33} + \tilde{\omega}^2 = 0 \\ {}^{5}R_{44} + 2X_{f4}{}^{d}\wp^{\,f}_{[d|||4]} &= {}^{4}R_{44} + \left[E_0 M_0 + E_0 B_0 + E_0 C_0 \right] + E_1 \frac{1}{\rho} \rho_{|1} = {}^{4}R_{44} - \tilde{\omega}^2 = 0 \end{aligned} \tag{3.9} $$

in accordance with (2.24). We retrieve the results of the 4-dimensional theory. Thus, we have shown that an embedding of the model into a 5-dimensional flat space is possible. This will facilitate discovering an interior solution for the NUT metric.

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[4] The projectors transform the 4-dimensional pseudo hypersphere embedded into a 5-dimensional flat space into the surface of the NUT geometry.

4. THE SEED METRIC

With the intrinsic transformation

$$\Delta_1^{1'} = \Delta_2^{2'} = \Delta_3^{3'} = \alpha_D, \quad \Delta_4^{4'} = a_D, \quad \alpha_D = \frac{A}{r}, \quad a_D = \frac{r}{A} \tag{4.1}$$

operating on the tetrads[5]

$$\overset{m}{e}_i = \Delta_i^{i'} \overset{m}{e}_{i'}, \quad \underset{m}{e}^i = \Delta_{i'}^{i} \underset{m}{e}^{i'}, \qquad \partial_1 = a_G a_D \frac{\partial}{\partial r} \tag{4.2}$$

we have deformed the rods and gauged the clocks and thus arranged the static seed metric. The new components of the tetrads are

$$\overset{1}{e}_1 = \alpha_D \alpha_G, \quad \overset{2}{e}_2 = \alpha_D r, \quad \overset{3}{e}_3 = \alpha_D r \sin\vartheta, \quad \overset{4}{e}_4 = a_D a_G \ . \tag{4.3}$$

With these tetrads and their inverse values we calculate the connexion coefficients, or alternatively with the intrinsic transformation

$$^*A_{[mn]}{}^s = A_{[mn]}{}^s + \Delta_{mn}{}^s, \quad \Delta_{mn}{}^s = \underset{m}{e}^{k'} \underset{n}{e}^{j'} \overset{s}{e}_{i'} \Delta_i^{i'} \Delta^i_{[j'|k']} \ .$$

The new connexion coefficients are

$$^*A_{mn}{}^s = A_{mn}{}^s + \Delta_{mn}{}^s + \Delta^s{}_{nm} + \Delta^s{}_{mn} \ . \tag{4.4}$$

If one deforms the metric a new quantity appears with only one component

$$D_1 = \frac{1}{\alpha_D} \alpha_{D|1} = -a_G \frac{l^2}{A^3} \ . \tag{4.5}$$

It enhances the field strengths of the reduced metric to the one of the seed metric

$$^*B_1 = B_1 + D_1, \quad ^*C_1 = C_1 + D_1, \quad ^*C_2 = C_2, \quad ^*E_1 = E_1 + D_1 \ . \tag{4.6}$$

In contrast to the reduced metric the 1-components of the field strengths of the seed metric

$$B_1 = a_D a_G \frac{1}{r}, \quad C_1 = a_D a_G \frac{1}{r}, \quad E_1 = a_D \frac{1}{\rho} \tan\varepsilon = -a_D \alpha_G \left(\frac{M}{r^2} + \frac{l^2}{r^3} \right) \tag{4.7}$$

have, in addition, the deformation factor a_D. A compressed notation

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[5] The primed indices refer to the reduced metric.

$$*B_1 = *C_1 = a_D^2 a_G \frac{1}{A} \tag{4.8}$$

is also possible. Facing the 5-dimensional representation of this model, we introduce the quantities

$$M_0 = \frac{a_D}{\rho}, \quad B_0 = a_D v_G \frac{1}{r}, \quad B_0 = a_D v_G \frac{1}{r}, \quad E_0 = -\frac{a_D}{\rho}, \quad D_0 = -v_G \frac{l^2}{A^3} \tag{4.9}$$

to provide a compact form of the field equations. v_G has been defined in (2.13). B, C, and D are horizontal quantities[6], and E a vertical one. It follows the relation

$$B_c E^c = C_c E^c = D_c E^c = 0 \tag{4.10}$$

that has to be considered in several calculations. The 0-components (marked by an asterisk) should also be constructed by the method (4.6). To make the appraisal of the field equations more transparent we list some auxiliary formulae[7]. For the graded derivatives one has to use the quantities with an asterisk (4.4). With some algebra, one obtains the Ricci

$$\begin{aligned} R_{mn} = &-\left[*B_{n\underset{2}{\|}m} + *B_n *B_m \right] - b_n b_m \left[*B^s{}_{\underset{2}{\|}s} + *B^s *B_s \right] \\ &-\left[*C_{n\underset{3}{\|}m} + *C_n *C_m \right] - c_n c_m \left[*C^s{}_{\underset{3}{\|}s} + *C^s *C_s \right] \; . \\ &+\left[*E_{n\underset{4}{\|}m} - *E_n *E_m \right] + u_n u_m \left[*E^s{}_{\underset{4}{\|}s} - *E^s *E_s \right] \end{aligned} \tag{4.11}$$

Resolving the subequations one obtains expressions that include the second fundamental forms of this geometry. For working up the last line of (4.11) the relation

$$E_1 \frac{1}{\rho} \rho_{|1} = -\left[E_0 M_0 + E_0 B_0 + E_0 C_0 \right] - \alpha_D^2 \omega^2 \tag{4.12}$$

is needed. By use of the enhanced monopole field strength

$$\mathcal{H}_{23} = -i a_D a_G \omega, \quad \mathcal{H}^2 = \mathcal{H}_{mn} \mathcal{H}^{mn} = 2 \mathcal{H}_{23} \mathcal{H}^{23} \tag{4.13}$$

the new Ricci

---

[6] They are located in the parallels of the surface

$B_1 D_1 = -a_G^2 \omega^2, \; *B_1 D_1 = -a_D^2 a_G^2 \omega^2 = H_{23} H_{23}, \; H_{23} = -i a_D a_G \omega, \; B_0 D_0 = -v_G^2 \omega^2, \quad *B_0 D_0 = -a_D^2 v_G^2 \omega^2, \; B_c D^c = -\omega^2$

[7] $B_0 B_0 = B_0 C_0 = \frac{v_G^2}{A^2}, \; *B_0 B_0 = *B_0 C_0 = a_D^2 \frac{v_G^2}{A^2}, \; D_1 D_1 = a_G^2 \omega^2 - a_D^2 a_G^2 \omega^2, \; B_0 E_0 = -2 M_0 B_0 = \frac{v_G^2}{A^2} + \alpha_D^2 \omega^2$

$2 E_1 D_1 = 2 M_0 D_0 = \alpha_D^2 \omega^2 - \alpha_G^2 \omega^2, \; 2 M_0 *B_0 + B_0 C_0 - B_1 D_1 = 0, \; *B_0 M_0 + *B_0 C_0 - *B_0 E_0 = \omega^2, \; *B_c D^c = -a_D^2 \omega^2$

$$R_{mn} = \begin{pmatrix} 0 & & & \\ & \mathcal{H}^2 & & \\ & & \mathcal{H}^2 & \\ & & & -\mathcal{H}^2 \end{pmatrix} \tag{4.14}$$

differs considerably from the reduced Ricci (2.24) as the [11]-component vanishes. The Ricci is not traceless

$$R = \mathcal{H}^2 \ . \tag{4.15}$$

Thus, the stress-energy tensor

$$\kappa T_{mn} = -2\left[\mathcal{H}_m{}^s \mathcal{H}_{ns} - \frac{1}{4} g_{mn} \mathcal{H}^2\right] + u_m u_n \mathcal{H}^2 \tag{4.16}$$

only to some extent complies with the stress-energy tensor of the electrodynamics. It is covariantly conserved

$$T^{mn}{}_{\|n} = 0 \ . \tag{4.17}$$

The Maxwell-like equations

$$\mathcal{H}^{mn}{}_{\underset{4}{\|n}} = 0, \quad \mathcal{H}_{<mn\underset{4}{\|}s>} + {}^*E_{<m} \mathcal{H}_{ns>} = 0 \tag{4.18}$$

are also valid. They may be written as

$$\text{rot}\vec{\mathcal{H}} = 0, \quad \text{div}\vec{\mathcal{H}} + {}^*\vec{E}\vec{\mathcal{H}} = 0 \ .$$

The second term of the last equations indicates the selfinteraction of the fields. From the [44]-component of (4.11) one reads the interesting relation

$$\text{div}\, {}^*\vec{E} = {}^*E^2 - \mathcal{H}^2 \ . \tag{4.19}$$

The right side of this equation is the energy density of the fields. According to the definition (4.13), the energy is positive. Lynden-Bell and Nouri-Zonoz [4] have found similar gravimagnetic field equations. Since they had used the co-ordinate method, their field quantities differ from ours by factors. The relation (4.19) can also be written as

$${}^*E^s{}_{\|s} = -\mathcal{H}^2 \ . \tag{4.20}$$

It is possible to extract the Schwarzschild-like force of gravity

$$\bar{E}^s{}_{\|s} = 0, \quad \bar{E}_1 = -a_D \alpha_G \frac{M}{r^2} \ . \tag{4.21}$$

It corresponds to (2.33) of the reduced model. For the third term in (4.7) one obtains by analogy with (2.32)

$$\mathcal{E}^{s}{}_{\|s} = a_D^2 \tilde{\omega}^2, \quad \mathcal{E}_1 = -a_D \alpha_G \frac{l^2}{r^3} \tag{4.22}$$

and finally

$$D^{s}{}_{\|s} = -\mathcal{H}^2 - a_D^2 \tilde{\omega}^2, \quad \mathcal{E}^{s}{}_{\|s} + D^{s}{}_{\|s} = -\mathcal{H}^2 \ . \tag{4.23}$$

We realize that the monopole energy density is the source of these quantities that deviate from the Schwarzschild-like force of gravity.

The last step to the proper NUT metric we perform with the help of an intrinsic transformation. From the time-like element of the NUT metric

$$dx^4 = a_D a_G \left[2il\left(1 - \cos\vartheta\right) d\varphi + dit \right] \tag{4.24}$$

we read the transformation coefficients

$$\begin{array}{lll} \Lambda^{3}_{3'} = 1, & \Lambda^{4}_{3'} = -2il\left(1-\cos\vartheta\right), & \Lambda^{4}_{4'} = 1 \\ \Lambda^{3'}_{3} = 1, & \Lambda^{4'}_{3} = \ \ 2il\left(1-\cos\vartheta\right), & \Lambda^{4'}_{4} = 1 \end{array} . \tag{4.25}$$

The tetrads are transformed according to

$$\overset{m}{e}_{i} = \Lambda^{i'}_{i} \overset{m}{\tilde{e}}_{i'}, \quad \underset{m}{e}^{i} = \Lambda^{i}_{i'} \underset{m}{e}^{i'} \tag{4.26}$$

whereby the co-ordinate indices of the formerly discussed seed metric are primed

$$A_{mn}{}^{s} = {}^*A_{mn}{}^{s} + \Lambda_{mn}{}^{s} + \Lambda^{s}{}_{nm} + \Lambda^{s}{}_{mn}, \quad \Lambda_{mn}{}^{s} = \underset{m}{e}^{k'} \underset{n}{e}^{j'} \overset{s}{\tilde{e}}_{i'} \Delta^{i'}_{i} \Lambda^{i}_{[j'|k']} \ . \tag{4.27}$$

Recasting this relation one obtains for the NUT metric

$$A_{mn}{}^{s} = {}^*A_{mn}{}^{s} + H_{mn}{}^{s}, \quad H_{mn}{}^{s} = \mathcal{H}_{mn} u^s + \mathcal{H}^{s}{}_{m} u_n + \mathcal{H}^{s}{}_{n} u_m \ . \tag{4.28}$$

The coefficients of the seed metric *A are enhanced with expressions of the monopole field strength. Working out the Ricci, one obtains the vacuum field equations

$$R_{mn} = {}^*R_{mn} + 2u_{(m}\mathcal{H}^{s}{}_{n)\underset{4}{\|}s} - 2\mathcal{H}_{m}{}^{s}\mathcal{H}_{ns} + u_m u_m \mathcal{H}^2 = 0 \ . \tag{4.29}$$

The stress-energy tensor of the seed metric is nullified by the intrinsic transformation. The relation

$$\mathcal{H}^{s}{}_{n\underset{4}{\|}s} = 0 \tag{4.30}$$

decouples from the field equations. We are left with

$${}^*R_{mn} = 2\mathcal{H}_{m}{}^{s}\mathcal{H}_{ns} - u_m u_n \mathcal{H}^2 \tag{4.31}$$

in accordance with (4.14).

It is worth to note that Lynden-Bell and Nouri-Zonoz [4] tried to ascribe physical meaning to the gravitational monopoles. In addition, Sackfield [5] searched for their explanations. Bonnor [6] worked out the basic properties of the NUT metric. He argued that in contrast to the Schwarzschild metric the NUT metric has no singularity at r = 0 but there are changes in the sign of the radial and time-like elements of the metric due to the factors $a_D a_G$. The roles of space and time are interchanged in the negative range of these factors. The event horizon is located at $r_H = M \pm \sqrt{M^2 + l^2}$. The force of gravity is infinite at r_H. Bonnor assumed a singularity at $\vartheta = 0$ and $\vartheta = \pi$. The latter one should be a physical one and therewith he explained the source of the fields as a massless rotating rod. Manko and Ruiz [7] objected to Bonnor and interpreted the source as two semi-infinite counter-rotating rods, separated by a massless region. It seems to us that too much relevancy has been ascribed to the ϑ-problem in the literature. The field quantities, derived from the NUT metric, formally do not differ from the ones of the Schwarzschild field strengths and from the ones of comparable models. The singularities appearing at $\vartheta = 0$ and $\vartheta = \pi$ are the well-known co-ordinate singularities of the polar system and can be arbitrarily displaced. One has to bear in mind that the components of the metric are co-ordinate objects and do not to enunciate physical objects in a proper way. Therefore the singularities are deficiencies of description and do not have any physical meaning. Moreover, the monopole field is independent of the angle ϑ. Since the two special ϑ-values alter the metric, it was assumed that the model is not flat at infinity. Taking a glance at the field strengths, one can prove that they vanish at infinity.

A 5-dimensional formulation of the NUT metric is not expedient. A satisfactory formulation is anticipated by the 5-dimensional representation of the reduced model. The intrinsic transformation, transforming the reduced metric into the seed metric is equivalent to a gauging of the rods and clocks that measure the quantities on the surface. The surface itself is invariant under this transformation. The second intrinsic transformation that implements the rotational content has no influence on the geometrical properties of the surface.

Some more authors have treated the NUT problem: Ashtekar and Sen [8], Bossard, Nicolai and Stelle [9], Dowker and Roche [10], Gautreau and Hoffman [11], Mena and Natario [12], Miller Kruskal and Godfrey [13], Moncrieff [14], Neugebauer and Kramer [15]. Nouri-Zonoz [16], Reina and Treves [17] demonstrated that the combined Kerr-Newman-NUT solution can be derived with the help of complex potential formalism, introduced by Ernst. Kinnersley [18] refined some models related to the NUT metric with the help of the Newman-Penrose formalism. Misner [19] sounded out the singularities of the NUT metric and explicitly calculated the Ricci. Miller [20] analyzed the singularities and the maximal analytic extension. Tomimatsu and Kihara [21] investigated the properties of the symmetry axis of a metric that is the superposition of two combined Kerr-NUT metrics. Wei [22] and Yamazaki [23] extended the combined Kerr-NUT metric to a charged one.

5. CONCLUSIONS

We have shown that the NUT metric can be interpreted geometrically by a surface into a 5-dimensional flat space. Since the NUT metric is closely related to the Schwarzschild metric this surface is similar to the one of the Schwarzschild model which we have discussed in previous papers. Two intrinsic transformations are needed to explain the geometrical structure of the NUT metric. These embedding techniques will offer us the advantage of enabling us to derive an interior for the NUT metric that reduces to the Schwarzschild interior by putting zero the NUT parameter. We will show this in a subsequent paper.

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