ARG Austrian Reports on Gravitation

ARG-2013-03

REMARKS ON THE MODEL OF OPPENHEIMER AND SNYDER III

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Keywords: gravitational collapse, field strengths, field equations, comoving and non-comoving co-ordinate system, transformation of systems

Contents

We calculate the field strengths of the Oppenheimer-Snyder model with the help of tetrads and Ricci-rotation coefficients. We investigate the transformation of the field strengths and the field equations from the comoving to the non-comoving reference system.

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1. INTRODUCTION

In previous papers [1,2] we have thoroughly re-investigated the model of Oppenheimer and Snyder [3] which describes the pressure-free collapse of a stellar object. We have shown that the star has infinite extent and vanishing density at the beginning of the collapse. It is environed by the Schwarzschild field and it collapses in free fall, whereas the surface of the star has the velocity of an object which comes from infinity. The collapse velocity would reach the speed of light at the event horizon. There the gravitational forces would be infinitely large.

Although the model of OS is physically unrealistic we make further investigation, because the model is sufficiently interesting and perhaps mathematically formal structures can serve as a template for models that describe Nature better. In particular, we examine the field equations in the comoving and in the non-comoving systems in compliance with a notation as compact as possible and maintaining strict covariance. By means of a Lorentz transformation which we perform in blocks on the field equations of both systems we transform the field equations into one another. Finally, we note the equations of motion for the particles which are subjected to the collapse and we note the conservation laws.

2. THE FIELD STRENGTHS

In previous papers we have already simplified the representation of the OS model so that a direct path is leading us to the field strengths, whereby we have used exclusively the tetrad representation. First, we write down the line element in the comoving system for the interior of a stellar object

$$(\mathrm{A})\quad ds^2 = K^2\left[dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta\, d\varphi^2\right] + dit'^2 \ . \qquad (2.1)$$

Therein K is the *scale factor* which establishes the connection of the non-comoving radial co-ordinate r with the comoving r'

$$r = K(t')r' \ . \qquad (2.2)$$

From (2.1) can be seen that the OS line element appears to be flat, but is time-dependent. Since the collapsing star is to be surrounded outside by the static Schwarzschild field, the geometry of the interior leads to linking problems on the boundary, which will be discussed in the next Section: A time-dependent and a time-independent solution are to be reconciled.

The metrical factor before the time-like element of the metric indicates that the surface of the star is in free fall from infinity. From the metric (2.1) we read the 4-bein

$$(\mathrm{A})\quad \overset{1'}{e}_{1'} = K, \quad \overset{2'}{e}_{2'} = K r', \quad \overset{3'}{e}_{3'} = K r' \sin\vartheta, \quad \overset{4'}{e}_{4'} = 1 \ . \qquad (2.3)$$

For the non-comoving system the metric is

$$(B)\quad ds^2=\alpha^2 dr^2+r^2 d\vartheta^2+r^2\sin^2\vartheta\, d\varphi^2+a_T^2 dit^2$$

$$\alpha=\frac{1}{\sqrt{1-\frac{r^2}{\mathcal{R}_g^2}}},\quad a_T=\alpha\sqrt{\frac{r_g}{2M}}\frac{y-1}{y^{3/2}},\quad y=\frac{1}{2}\left[\left(\frac{r'}{r'_g}\right)^2-1\right]+\frac{r'_g}{2M}\frac{r}{r'},\quad \alpha_T=1/a_T\quad . \tag{2.4}$$

Therein r_g is the value of r on the surface of the stellar object. The corresponding value in the comoving system is the constant quantity r'_g . From the viewpoint of the comoving observer the position of the surface does not change.

$$\mathcal{R}_g=\frac{\rho_g}{2}=\sqrt{\frac{r_g^3}{2M}} \tag{2.5}$$

is equal to half the radius of the curvature of the Schwarzschild parabola on the boundary surface. The complicated structure of y is probably due to the fact that the OS model does not smoothly adjoin the Schwarzschild environment in a simple manner. Fortunately[1], however, one has

$$y_{|1}=0\ . \tag{2.6}$$

The 4-bein corresponding to (B) is

$$(B)\quad \overset{1}{e}_1=\alpha,\quad \overset{2}{e}_2=r,\quad \overset{3}{e}_3=r\sin\vartheta,\quad \overset{4}{e}_4=a_T\quad . \tag{2.7}$$

Derivating the 4-bein we build the Ricci-rotation coefficients for both systems, to which we assign the structure

$$A_{mn}{}^s=U_{mn}{}^s+B_{mn}{}^s+C_{mn}{}^s\ . \tag{2.8}$$

The radial component U and the two lateral quantities B and C are further decomposed

$$U_{mn}{}^s=h_m{}^sU_n-h_{mn}U^s,\quad B_{mn}{}^s=b_mB_nb^s-b_mb_nB^s,\quad C_{mn}{}^s=c_mC_nc^s-c_mc_nC^s\ . \tag{2.9}$$

Therein is

$$h_{mn}=\begin{pmatrix}1&&&\\&0&&\\&&0&\\&&&1\end{pmatrix} \tag{2.10}$$

a submatrix of the metric and the

$$b_m=\{0,1,0,0\},\quad c_m=\{0,0,1,0\} \tag{2.11}$$

are the unit vectors in the lateral 2- and 3-directions of the spherical co-ordinate system. The associated quantities

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[1] We use the following notation: A prime on a kernel of a quantity indicates a comoving quantity of the system (A), a prime on the index of a quantity indicates that it is measured in (A). For example $\Phi_{m'}$ is a quantiy of the non-comoving system (B) measured in (A), $'\Phi_m$ is a quantiy of (A) measured in (B).

$$B_m = \left\{\frac{a}{r},0,0,0\right\}, \quad C_m = \left\{\frac{a}{r},\frac{1}{r}\cot\vartheta,0,0\right\}, \quad a = 1/\alpha \tag{2.12}$$

are easy to calculate from (2.7). Since the lateral part of the line element is invariant with respect to the transition from (B) to (A) by means of the relation (2.2) the values (2.11) also apply to $b_{m'}$ and $c_{m'}$. OS specify a transformation between comoving and non-comoving co-ordinates. In [1] we have associated the co-ordinate transformation with a reference frame transformation which causes a pseudo-rotation of the 4-bein. Therefore this transformation is the Lorentz transformation

$$L^1_{1'} = \alpha, \quad L^1_{4'} = -i\alpha v, \quad L^4_{1'} = i\alpha v, \quad L^4_{4'} = \alpha \ . \tag{2.13}$$

The Lorentz factor of this transformation is identical with the radial metrical factor of the metric (B) in (2.4). This means that the state of motion of the particles is determined by the geometry inside the collapsing object. The velocity of the particles can be read from the Lorentz factor and is

$$v = -\frac{r}{\mathcal{R}_g} \ . \tag{2.14}$$

It is equal to zero at the center of the star $(r = 0)$, and on the surface $(r = r_g)$ it is

$$v_g = -\sqrt{\frac{2M}{r_g}} \tag{2.15}$$

in accordance with (2.5).

v_g can be considered as the contraction velocity of the star and corresponds to the speed of an object in free fall coming from infinity in the Schwarzschild field. Therefore, the surface of the star must also come from the infinite.

The lateral quantities (2.12) are vectors and transform homogeneously into the comoving system

$$\begin{aligned} B_{m'} &= L^m_{m'} B_m = \{\alpha B_1, 0, 0, -i\alpha v B_1\} = \left\{\frac{1}{r},0,0,\frac{i}{\mathcal{R}_g}\right\} \\ C_{m'} &= L^m_{m'} C_m = \{\alpha C_1, C_2, 0, -i\alpha v C_1\} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0,\frac{i}{\mathcal{R}_g}\right\} \end{aligned} \ . \tag{2.16}$$

It can be seen that the space-like parts of the two quantities correspond to a flat geometry. This should be investigated in more detail.

However, the sub-variables of (2.8) contained in the quantities U transform inhomogeneously. In general, one has for the Riccci-rotation coefficients

$$\begin{aligned} {}'A_{m'n'}{}^{s'} &= L^{m\ n\ s'}_{m'n's} A_{mn}{}^{s} + {}'L_{m'n'}{}^{s'} \\ A_{mn}{}^{s} &= L^{m'n'\ s}_{m\ n\ s'} {}'A_{m'n'}{}^{s'} + L_{mn}{}^{s} \end{aligned} \tag{2.17}$$

We call each of the second terms Lorentz terms. They are defined as

$$ {}'L_{m'n'}{}^{s'} = L^{s'}_{s} L^{s}_{n'|m'}, \quad L_{mn}{}^{s} = L^{s}_{s'} L^{s'}_{n|m}, \quad {}'L_{m'n'}{}^{s'} = -L^{m\ n\ s'}_{m'n's} L_{mn}{}^{s} \ . \tag{2.18} $$

In addition, if one writes using (2.13)

$$ \begin{gathered} {}'L_{4'1'}{}^{4'} = {}'L_{1'} = i\alpha^2 v_{|4'} \quad {}'L_{1'4'}{}^{1'} = {}'L_{4'} = -i\alpha^2 v_{|1'} \\ L_{41}{}^{4} = L_{1} = -i\alpha^2 v_{|4} \quad L_{14}{}^{1} = L_{4} = i\alpha^2 v_{|1} \end{gathered} \ , \tag{2.19} $$

one can bring the Lorentz terms into the form

$$ {}'L_{m'n'}{}^{s'} = h_{m'}{}^{s'}\, {}'L_{n'} - h_{m'n'}\, {}'L^{s'}, \quad L_{mn}{}^{s} = h_{m}{}^{s} L_{n} - h_{mn} L^{s} . \tag{2.20} $$

With this arrangement one obtains from (2.9), first relation, and from (2.17)

$$ {}'U_{m'} = U_{m'} + {}'L_{m'}, \quad U_{m'} = L^{m}_{m'} U_{m}, \quad U_{m} = {}'U_{m} + L_{m}, \quad {}'U_{m} = L^{m'}_{m}\, {}'U_{m'} \ . \tag{2.21} $$

According to (2.18) one has to take into account

$$ L_{m} = -L^{m'}_{m}\, {}'L_{m'} \ . \tag{2.22} $$

The relation (2.21) simplifies the conversion of the field quantities of the field system (A) into the ones of the system (B) and vice versa.

Now the quantities U and L are to be calculated. For this reason we need some relations which we have noted in [1,2]

$$ \frac{\partial r_g}{\partial r'} = 0, \quad \frac{\partial r_g}{\partial t'} = v_g, \quad \frac{\partial \mathcal{R}_g}{\partial r'} = 0, \quad \frac{\partial \mathcal{R}_g}{\partial t'} = -\frac{3}{2} \ . \tag{2.23} $$

With (2.7) and then with (2.13) we first obtain

$$ r_{|m} = \{a, 0, 0, 0\}, \quad r_{|m'} = \{1, 0, 0, -iv\} . \tag{2.24} $$

From the second formula we derive the relations

$$ \frac{\partial r}{\partial t'} = v, \quad \frac{dx^1}{dT} = v \ , \tag{2.25} $$

wherein the second item is the Lorentz conformal expression for the velocity. We have used $dx^1 = \alpha dr$ and the Lorentz relation $dT / dT' = \alpha$. Furthermore, we have to take into account that in the comoving co-ordinate system the co-ordinate time t' coincides with the proper time T'.

From (2.14) we now can calculate the changes of the velocity which enter into the field strengths and Lorentz terms

$$ \begin{gathered} v_{|m} = \{1, 0, 0, 0\}\left(-\frac{a}{\mathcal{R}_g}\right) + \{-i\alpha v, 0, 0, \alpha\}\left(-3iv\frac{1}{\rho_g}\right) \\ v_{|m'} = \{\alpha, 0, 0, -i\alpha v\}\left(-\frac{a}{\mathcal{R}_g}\right) + \{0, 0, 0, 1\}\left(-3iv\frac{1}{\rho_g}\right) \end{gathered} \ . \tag{2.26} $$

Both quantities include a circular and a parabolic component. We will come back to it. With

$$ \frac{1}{\alpha}\alpha_{|m} = \alpha^2 v\, v_{|m} \tag{2.27} $$

several field strengths and the Lorentz terms can be calculated. We start with

$$'U_{4'} = 'U_{1'4'}{}^{1'} = -\overset{1'}{e}_{1'}\, e^{1'}_{1'|4'} = \frac{1}{\mathcal{K}}\mathcal{K}_{|4'} \ . \tag{2.28}$$

Thus, the quantity 'U describes the change of the scale factor. On the other hand we have by (2.2) and (2.24)

$$r_{|4'} = \mathcal{K}_{|4'} r' = -iv = i\frac{r}{\mathcal{R}_g}$$

and hence

$$'U_{4'} = \frac{i}{\mathcal{R}_g}, \quad \frac{1}{\mathcal{K}}\mathcal{K}^{\bullet} = -\frac{1}{\mathcal{R}_g}, \quad \mathcal{K}^{\bullet} = \frac{\partial \mathcal{K}}{\partial t'} \ . \tag{2.29}$$

Together with (2.16) we obtain three kindred quantities

$$'U_{4'} \overset{*}{=} B_{4'} \overset{*}{=} C_{4'} = \frac{i}{\mathcal{R}_g} \ . \tag{2.30}$$

The standard expression of the 'expansion' results with $'u_{n'} = \{0,0,0,1\}$ in

$$'u^{n'}{}_{||n'} = 'A_{n'}\,'u^{n'} = 'U_{4'} + 'B_{4'} + 'C_{4'} = 3\frac{i}{\mathcal{R}_g} \tag{2.31}$$

and is here to be interpreted as a spatially uniform contraction which has the same value at all points in space. Nothing changes if one arbitrarily rotates or shifts the local co-ordinate system. The model has spherical symmetry.

According to (2.3) the metrical factor of the time-like part of the line element (A) is equal to 1, indicating the free fall from infinity. Thus, no other forces can be derived from the metric (A), in particular, no effect of gravity. Therefore one has

$$'U_{m'} = \left\{0,0,0,\frac{i}{\mathcal{R}_g}\right\} \ . \tag{2.32}$$

The calculation of the field quantities in the non-comoving system is more complex. The radial metrical factor of the metric (B), the curvature quantity α, is according to (2.2) dependent on the time. With (2.27) and (2.26) one has

$$U_4 = A_{14}{}^1 = -\overset{1}{e}_1\, e^1_{1|4} = \frac{1}{\alpha}\alpha_{|4} = \alpha^2 v v_{|4} = -3i\alpha^3 v^2 \frac{1}{\rho_g} \ . \tag{2.33}$$

Using (2.6) one gets from (2.4)

$$U_1 = A_{41}{}^4 = -\overset{4}{e}_4\, e^4_{4|1} = \frac{1}{a_T} a_{T|1} = \frac{1}{\alpha}\alpha_{|1} + \frac{1}{\sqrt{\frac{r_g}{2M}}}\left(\sqrt{\frac{r_g}{2M}}\right)_{|1} \ . \tag{2.34}$$

Therin is

$$\frac{1}{\alpha}\alpha_{|1} = E^R_1 - 3\alpha^3 v^3 \frac{1}{\rho_g}$$

where

$$E_1^R = -\alpha v \frac{1}{\mathcal{R}_g} \tag{2.35}$$

is a positive quantity which lies in the radial direction. Usually it occurs by the evaluation of the spherical geometry, but cannot be identified as gravity inside the stellar object. The second term in (2.34) yields the negative contribution

$$E_1^S = \alpha v \frac{1}{\rho_g} \ . \tag{2.36}$$

The quantity formally corresponds to the gravity of the exterior Schwarzschild field. To calculate this expression the relation $r_{g|1} = \alpha v\, v_g$ derived in [2] was used. Combining the quantities one obtains

$$U_1 = -E_1^R - 3\alpha^3 v \frac{1}{\rho_g} \ .$$

Finally, we have with

$$U_m = \left\{ -E_1^R - 3\alpha^3 v \frac{1}{\rho_g}, 0, 0, -3i\,\alpha^3 v^2 \frac{1}{\rho_g} \right\} \tag{2.37}$$

and the quantities (2.12) completely computed the field strengths of the system (A).

3. THE GEOMETRY

Before we formulate Einstein's field equations with the help of the field quantities just obtained, we will investigate their behavior on the surface of the stellar object, ie at the boundary surface between the interior and exterior solutions. Nariai and Tomita [4, 5] have treated the problem of the linking conditions. They believed that the interior OS solution is not properly matched to the exterior Schwarzschild solution and have replaced the Schwarzschild solution by a more complicated one while maintaining the interior OS solution. Nariai [6] has discussed general linking conditions for models that consist of interior and exterior solutions. About the linking conditions between two areas O'Brien and Synge [7] have written in detail.

We are critical about these studies. Generally it is assumed that the metric coefficients and their first derivatives coincide on the boundary surface of the interior and exterior solutions. First, we investigate the metric coefficients. We have already computed the velocity at the boundary with (2.15). Therefore, α takes the Schwarzschild value in (2.4) and (2.7)

$$\alpha_g = \frac{1}{\sqrt{1 - \frac{2M}{r_g}}} \ .$$

In (2.4) becomes $y_g = r_g/2M$ and thus is $a_T^g = \sqrt{1-2M/r_g}$. The metric (2.7) coincides with the Schwarzschild metric on the surface of the star. The two spaces are connected.

The derivatives of the metric coefficients correspond to the field strengths presented by us. One can easily see from (2.12) that the quantities B and C take Schwarzschild values [8] on the boundary surface. Certainly, this does not apply to the quantities of (2.37). This can be understood quite well. The exterior solution remains static due to the Birkhoff theorem, whereas the interior solution collapses. Judging again the relations (2.33) - (2.37) one realizes that instead of (2.37) one can also write

$$U_m = -E_m^S + U_m^C, \quad E_m^S = \left\{\alpha v \frac{1}{\rho_g}, 0, 0, 0\right\}, \quad U_m^C = -\left\{-i\alpha v, 0, 0, \alpha\right\} 3i\alpha^2 v^2 \frac{1}{\rho_g} \ . \qquad (3.1)$$

E_m^S is the very quantity which takes the Schwarzschild value on the boundary surface. The term U_m^C does not arise straight away from the geometry, but from its change. According to (2.23), last expression, it contains the relation

$$\frac{1}{\mathcal{R}_g} \mathcal{R}_{g|4'} = 3i \frac{1}{\rho_g} \ .$$

A look at the field strengths of the comoving system immediately shows that the field strengths of the interior and exterior solutions match on the boundary surface. The exterior field is described by a freely falling observer. In [1] we have written down the relation between the OS co-ordinates and the comoving Lemaître co-ordinates and have discussed the free fall in detail in [8].

According to these considerations we are able to interpret the geometry of the OS model. The metric (B), whose structure is known from other models, suggests to interpret the interior spatial geometry as a spherical cap which is placed at a suitable position from the bottom of the Flamm's paraboloid up to the Schwarzschild geometry. $\mathcal{R}_g$ is the radius the cap, and we use the relation

$$r = \mathcal{R}_g \sin\eta \,, \qquad (3.2)$$

where η is the polar angle of the cap. According to the metric (B) is $\alpha = 1/\cos\eta$ and with (3.2) is $dr = \mathcal{R}_g \cos\eta \, d\eta$. We have for the metric (B)

$$\mathcal{R}^2 d\eta^2 + \mathcal{R}^2 \sin^2\eta \, d\vartheta^2 + \mathcal{R}^2 \sin^2\eta \sin^2\vartheta \, d\varphi^2 + a_T^2 dit^2 \ . \qquad (3.3)$$

Thus, one has found a simple geometric interpretation of the OS-model. During the collapse the spherical cap slides down on Flamm's paraboloid of the Schwarzschild environment. While the spherical cap shrinks during this process Flamm's paraboloid remains unchanged. This is also a very insightful presentation of the Birkhoff theorem. A collapse has no effect on an observer located in the exterior field. It can also be seen that the spherical cap cannot shrink beneath the event horizon. It would uncouple from Flamm's paraboloid. Thus, the OS model does not allow the formation of black holes. Mitra [9] has brought forward another argument for this subject.

We find that it is sufficient for the linking condition if the geometric basic structures match on the boundary surface. Additional terms which are not explained by the geometry of the spherical cap are ascribed to the collapse.

It is now to be clarified why the metric (A) appears to be flat. A reference-system transformation, ie, a transition from an observer being in rest to a comoving observer is not expected to change the geometric structure of the space. The flat appearance of the metric arises because the motion of the freely falling system is tightly coupled to the geometric structure of the model. This will now be examined in more detail. We consider the flat 5-dimensional embedding space, and at first we suppress all dimensions except the first two ones. The global Cartesian co-ordinates in this space are $\left\{x^{0''}, x^{1''}\right\}$ with $x^{0''}$ as the extra dimension and the local co-ordinates are $\left\{x^{0}, x^{1}\right\}$. The relations between the reference systems provide local rotations

$$\begin{aligned} dx^0 &= dx^{0''}\cos\eta - dx^{1''}\sin\eta \\ dx^1 &= dx^{0''}\sin\eta + dx^{1''}\cos\eta \end{aligned} .$$

Since the radial arc element of the line element (A) lies entirely in the local 1-direction

$$\begin{aligned} & dx^0 = 0, \quad dx^{0''} = \tan\eta\, dx^{1''}, \quad dx^{1''} = dr \\ & dx^{1^2} = dx^{0''^2} + dx^{1''^2} = \tan^2\eta\, dr^2 + dr^2 = \frac{1}{\cos^2\eta} dr^2 \end{aligned}$$

applies. According to (3.2) on has

$$\sin\eta = \frac{r}{\mathcal{R}_g}, \quad \cos\eta = \sqrt{1 - \frac{r^2}{\mathcal{R}_g}} = a = \frac{1}{\alpha} \ , \qquad (3.4)$$

whereby the radial part of the metric (A) is geometrically deepened. The parallels of the spherical cap have the radius r, and its curvature is defined by

$$B_{a''} = \left\{0, \frac{1}{r}, 0,0,0\right\}, \quad a'' = 0'',1'',\ldots,4'' \ . \qquad (3.5)$$

In the local system it has the form

$$B_a = \left\{\frac{1}{r}\sin\eta, \frac{1}{r}\cos\eta, 0,0,0\right\}, \quad a = 0,1,\ldots,4 \ . \qquad (3.6)$$

Using the Lorentz angle the Lorentz transformation (2.13) can be written as

$$L^{1}_{1'} = \cos i\chi, \quad L^{1}_{4'} = \sin i\chi, \quad L^{4}_{1'} = -\sin i\chi, \quad L^{4}_{4'} = \cos i\chi \qquad (3.7)$$

and is a pseudo-rotation through the imaginary angle $i\chi$ in the local [1,4]-plane. If one operates herewith on (3.6), one obtains with

$$B_{a'} = \left\{\frac{1}{r}\sin\eta, \frac{1}{r}\cos\eta\cos i\chi, 0,0, \frac{1}{r}\cos\eta\sin i\chi\right\} \qquad (3.8)$$

an expression whose 1-component will appear flat because the radial metric coefficient in (B) corresponds to the Lorentz factor. One has

$$\cos\eta\cos i\chi = 1 \ . \qquad (3.9)$$

From (2.14) and (3.4) one gathers

$$v = -\sin\eta, \quad a = \cos\eta$$

and one obtains

$$B_{a'} = \left\{ \frac{1}{\mathcal{R}_g}, \frac{1}{r}, 0, 0, \frac{i}{\mathcal{R}_g} \right\} \ . \tag{3.10}$$

The quantity $B_{a'}$ is generated from (3.5) by two transformations. A rotation in the [0,1]-plane and a pseudo-rotation in the [1',4']-plane. It should also be noted that this deceptive appearance also occurs in the exterior Schwarzschild field concerning the free fall. The Schwarzschild metric can be brought with a Lemaître transformation into a form which appears flat. This is only valid for the free fall from infinity. If the body starts at a finite position, the metric takes on a much more complicated form.

The metric of the comoving OS model is obtained by means of a Lemaître-like transformation which we have discussed in detail in [1]

$$g_{i'k'} = \Lambda^{i\ k}_{i'k'} g_{ik}, \quad \Lambda^{i}_{i'} = \overset{m'}{e}_{i'} L^{m}_{m'} \underset{m}{e}^{i} \ .$$

A co-ordinate transformation can never change the geometrical or physical background of the model, so that one cannot make any direct conclusions about the geometric structure of the model by means of the apparently flat representation (A). The field quantities, however, show that the basic structure of the geometrical model is invariant under a transformation of the co-ordinates and under a Lorentz transformation as well.

4. FIELD EQUATIONS

After these preliminaries we are able to investigate the transformation properties of the field equations. We start with the Ricci

$$R_{mn} = A_{mn}{}^{s}{}_{|s} - A_{n|m} - A_{rm}{}^{s} A_{sn}{}^{r} + A_{mn}{}^{s} A_{s} \tag{4.1}$$

and perform the decomposition according to (2.8) - (2.10). One has

$$\begin{aligned} R_{mn} = &- \left[U^{s}{}_{\underset{1}{||}s} + U^{s} U_{s} \right] h_{mn} \\ &- \left[B_{n\underset{2}{||}m} + B_{n} B_{m} \right] - b_{n} b_{m} \left[B^{s}{}_{\underset{2}{||}s} + B^{s} B_{s} \right] \\ &- \left[C_{n\underset{3}{||}m} + C_{n} C_{m} \right] - c_{n} c_{m} \left[C^{s}{}_{\underset{3}{||}s} + C^{s} C_{s} \right] \end{aligned} \ . \tag{4.2}$$

Therein we have used the graded derivatives [8]

$$U_{m\underset{1}{||}n} = U_{m|n}, \quad B_{m\underset{2}{||}n} = B_{m|n} - U_{nm}{}^{s} B_{s}, \quad C_{m\underset{3}{||}n} = C_{m|n} - U_{nm}{}^{s} C_{s} - B_{nm}{}^{s} C_{s} \ . \tag{4.3}$$

The field equations in the comoving system have the same form

$$R_{m'n'} = -\left['U^{s'}{}_{\underset{1}{||}s'} + 'U^{s'}\,'U_{s'} \right] h_{m'n'}$$
$$- \left[B_{n'\underset{2}{||}m'} + B_{n'}B_{m'} \right] - b_n b_m \left[B^{s'}{}_{\underset{2}{||}s'} + B^{s'}B_{s'} \right]. \tag{4.4}$$
$$- \left[C_{n'\underset{3}{||}m'} + C_{n'}C_{m'} \right] - c_n c_m \left[C^{s'}{}_{\underset{3}{||}s'} + C^{s'}C_{s'} \right]$$

The graded derivatives as well

$$'U_{m'\underset{1}{||}n'} = 'U_{m'|n'}, \quad B_{m'\underset{2}{||}n'} = B_{m'|n'} - 'U_{n'm'}{}^{s'}B_{s'}, \quad C_{m'\underset{3}{||}n'} = C_{m'|n'} - 'U_{n'm'}{}^{s'}C_{s'} - B_{n'm'}{}^{s'}C_{s'} \ . \tag{4.5}$$

However, the field quantities are defined differently for both systems, and partly transform inhomogeneously into each other. Now we use the results of the previous Section. With (2.21) and (2.22) the relation

$$'U^{s'}{}_{\underset{1}{||}s'} + 'U^{s'}\,'U_{s'} = U^{s}{}_{\underset{1}{||}s} + U^{s}U_{s} \tag{4.6}$$

can be derived. On the basis of

$$L^{n'm'}_{n\ m} B_{n'|m'} = B_{n|m} - L_{mn}{}^{s}B_{s}, \quad U_{mn}{}^{s} = L^{'m'n's}_{m\ n\ s'}\, 'U_{m'n'}{}^{s'} + L_{mn}{}^{s}$$

results

$$L^{n'm'}_{n\ m}\left[B_{n'|m'} - 'U_{m'n'}{}^{s'}B_{s'} + B_{m'}B_{n'} \right] = B_{n|m} - U_{mn}{}^{s}B_{s} + B_{m}B_{n}$$

and a corresponding relation for the quantity for C. The subequations of the Ricci are all invariant. Thus, the Ricci is invariant as well

$$L^{m'n'}_{m\ n} R_{m'n'} = R_{mn} \ . \tag{4.7}$$

With (2.29) and (2.23) we obtain

$$'U^{s'}{}_{\underset{1}{||}s'} + 'U^{s'}\,'U_{s'} = \frac{1}{2}\frac{1}{\mathcal{R}_g^2} \ ,$$

wherewith one can calculate the unprimed relation with (4.6), too. To evaluate the B- and C-equations in both systems similar derivations in [9] can serve as a support. Ultimately, one has for the Ricci

$$R_{\alpha'\beta'} = \frac{3}{2} g_{\alpha'\beta'} \frac{1}{\mathcal{R}_g^2}, \quad \alpha' = 1,2,3, \quad R_{\alpha'4'} = R_{4'\beta'} = 0, \quad R_{4'4'} = -\frac{3}{2}\frac{1}{\mathcal{R}_g^2}$$

and for the invariant measure of curvature

$$R = \frac{3}{\mathcal{R}_g^2} \ .$$

From this follows the Einstein tensor

$$G_{\alpha'\beta'} = 0, \quad G_{\alpha'4'} = G_{4'\beta'} = 0, \quad G_{4'4'} = -\frac{3}{\mathcal{R}_g^2} \ , \tag{4.8}$$

so that with (2.5) one can read off

$$\kappa\mu_0 = \frac{3}{\mathcal{R}_g^2} = \frac{6M}{r_g^3} \ . \tag{4.9}$$

Such an expression commonly occurs in other gravitation models. However, the energy density μ_0 depends on the time. Since the OS model collapses in free fall from infinity ($r_g = \infty$) the stellar object fills an infinitely large space at the beginning of the collapse and has at $t' = 0$ vanishing density. The stress-energy-momentum tensor of the unprimed system can be calculated from the unprimed field equations or with

$$T_{mn} = L^{m'n'}_{m\ n} T_{m'n'} = \mu_0\, 'u_m\, 'u_n , \quad 'u_m = \{-i\alpha v, 0, 0, \alpha\} \ . \tag{4.10}$$

Thus, the conservation law results in

$$T_m{}^n{}_{||n} = 'u_m \left[\mu_{0|n}\, 'u^n + \mu_0\, 'u^n{}_{||n}\right] + \mu_0\, 'u_{m||n}\, 'u^n \ . \tag{4.11}$$

With the results of previous Sections it can be shown that the motion of the particles in the collapsing object is geodesic

$$'u_{m||n}\, 'u^n = 0 \ . \tag{4.12}$$

Therefore remains

$$\mu_{0|n}\, 'u^n + \mu_0\, 'u^n{}_{||n} = 0 \ . \tag{4.13}$$

The last term we have derived in (2.31). Thus, one has in accordance with (2.23)

$$\dot{\mu}_0 = \frac{3}{\mathcal{R}_g}\mu_0 \ . \tag{4.14}$$

Let's take a look at the equation of motion (4.12). After reshaping a little we get

$$(-i\alpha v)_{|4'} = -\alpha(\alpha U_1 + i\alpha v U_4)$$

and with $m = m_0\alpha$

$$\frac{dmv}{dt'} = -mU_{1'} \ . \tag{4.15}$$

On the other hand one has with (2.19)

$$(-i\alpha v)_{|4'} = -i\alpha^3 v_{|4'} = -\alpha\, 'L_{1'}$$

and with (2.21)

$$'L_{1'} + U_{1'} = 'U_{1'} \equiv 0 \ .$$

It can be seen that the force of gravity acting on the particles in the interior of the stellar object is canceled by the counter force 'L in the comoving system. A comoving observer is not exposed to gravity.

5. SUMMARY

The OS model fulfills all the formal requirements of a field theory based on the laws of general relativity. One can formulate vectorial forces for the comoving and the non-comoving systems which can be converted from one system to the other system. The equation of motion and the law of conservation are satisfied. The physical usability of the model has been questioned. A pressure-free star which fills the vastness of the universe and has vanishing density at the beginning of the collapse and leaves an empty space with a Schwarzschild field during the collapse behind it, does not exist.´

6. REFERENCES

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