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ASPECTS OF THE MACH-EINSTEIN DOCTRINE AND SOME GEOPHYSICAL APPLICATION (A HISTORICAL REVIEW)

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Abstract

The present authors have given a mathematical model of Mach's principle and of the Mach-Einstein doctrine about the complete induction of the inertial masses by the gravitation of the universe. The analytical formulation of the Mach-Einstein doctrine is based on Riemann's generalization of the Lagrangian analytical mechanics (with a generalization of the Galilean transformation) on Mach's definition of the inertial mass and on Einstein's principle of equivalence.

All local and cosmological effects, - which are postulated as consequences of Mach's principle by C. Neumann, Mach, Friedländer and Einstein - result from the Riemannian dynamics with the Mach-Einstein doctrine. In celestial mechanics it follows, in addition, Einstein's formula for the perihelion motion. In cosmology, the Riemannian mechanics yields two models of an evolutionary universe with the expansion lows $R \sim t$ or $R \sim t^2$.

In this paper, secular consequences of the Mach-Einstein doctrine are examined concerning palaeogeophysics and celestial mechanics. The research predicted secular decrease of the Earth's flattening and secular acceleration of the motion of the Moon and of the planets. The numerical values of this secular effect agree very well with the empirical facts. In all cases, the secular variation $\dot{\alpha}$ of the parameter α is the order of magnitude $\dot{\alpha} = -H_0\alpha$, where H_0 is the instantaneous value of the Hubble constant: $H_0 = (\dot{R}/R)_0 \approx (0.5 - 1.0).10^{-10} a^{-1}$. - The relation of the secular consequences of the Mach-Einstein doctrine to those of Dirac's hypothesis on the expanding Earth, and to Darwin's theory of tidal friction are also discussed.

Keywords: celestial mechanics; Dirac's hypothesis; Mach-Einstein doctrine; Mach's principle; palaeogeophysics

1. PERIHELION MOTION OF THE PLANETS

Treder (1972) has developed an analytical mechanics where the Galilean-Newtonian inertia is substituted by the inductive effect of the gravitational potential of all cosmic masses, as it is formulated in the so-called Mach-Einstein doctrine, being based on Mach's principle as Einstein sharpened it. This analytical representation of the Mach-Einstein doctrine is based on a generalization of the Lagrangian mechanics discovered by Riemann for velocity-dependent potentials where we eliminated the Newtonian kinetic energy in the Riemannian Lagrange function according to the Mach-Einstein doctrine - on the basis of the generalized relativity postulate (see Treder 1972).

Treder (1972) has shown that in the "inertia-free" Riemannian Lagrange function

$$L^{*} = \sum_{A>B}^{N} \frac{m_A m_B}{r_{AB}} f \left(1 + \frac{3}{2} \frac{\upsilon^2{}_{AB}}{c^2} \right), \tag{1}$$

for the $N >> 1$ point-like imagined particles P_a of the universe, the expression

$$m_a^{*} = \frac{3f}{r^2} m_a \sum_{B+a}^{N} \frac{m_B}{r_{AB}} = m_a \frac{f}{r^2} \left(\frac{9}{2} \frac{M}{R} + 3 \sum_{\alpha+a}^{a} \frac{m_\alpha}{r_{a\alpha}} \right) = \bar{m}_a^{*} + \Delta m_a^{*} \tag{2}$$

takes over the place of the inertial mass of the particles, where m_a, m_B, m_α, resp., are the gravitational masses (the gravitational charges) of the particles P_a. The decisive part $\bar{m}_a^{*}$ of this inertial mass is induced by the mid-collective gravitational potential of the universe

$$\Phi = \frac{3M}{2R} f \quad \left(\begin{matrix} M \text{ is the mass} \\ R \text{ is the radius} \end{matrix} \right\} \text{ of the cosmic particle cloud} \Bigg) \tag{3}$$

which can be supposed as being located independently in the central region of the cosmic particle cloud. Small corrections Δm_a^{*} of the inertial masses are then to be applied as postulated by Mach (1933), B. and I. Friedländer (1896) and by Einstein (1965, No. 23, 1969). They are induced by the inertial masses of bodies which are due to the gravitational potential of the n bodies P in the vicinity. These corrections are location dependent:

$$\Delta m_a^{*} = 3 m_a \frac{f}{c^2} \sum_{\alpha}^{n} \frac{m_\alpha}{r_{a\alpha}} . \tag{4}$$

Based on Mach's dynamic mass definition (1933) and the determination of the effective gravity constant f in the Newtonian gravitational law it follows that the gravity constant f and the gravitational charge m_a are so normalized that the following momentarily valid:

$$m_a^* = m_a \left(1 + 3 \frac{f}{c^2} \sum_{\alpha}^{n} \frac{m_\alpha}{r_{a\alpha}} \right) \tag{5}$$

$$m_a \frac{9}{2c^2} f \frac{M}{R} = m_a \tag{5a}$$

(see Treder 1972). Expression (5) corresponds to the dependence deduced by Einstein (1969) from the local gravitational potential on the basis of the equivalence and general relativity principles of the inertial masses (This was a restriction when choosing the constant 3/2 in the Lagrange function (1)).

The first consequence concerning celestial mechanics of the Riemann-Mach mechanics follows from Eq. (5) for the motion of a test particle P_1 (of mass m_1) around a central mass $m_2 = m$ with three equations of motion (Treder 1972):

$$\frac{d}{dt}\left[\left(1 + \frac{3fm}{c^2 r} \right) \upsilon^i \right] = \frac{\partial}{\partial x^i} \left(\frac{fm}{r} \left[1 + \frac{3\upsilon^2}{2c^2} \right] \right), \tag{6}$$

which gives for $n^2 << c^2, fm/r << c^2$

$$\frac{1}{r} = \frac{fm}{h^2} \left(1 + \varepsilon \cos\varphi \left[1 + \frac{3f^2m^2}{h^2c^2} \right] \right) \tag{7a}$$

i. e. the Kepler-motion is obtained with Einstein's perihelion rotation (1916).

$$\delta\varphi = 6\pi \frac{f}{c^2} \frac{m}{a\left(1 - e^2\right)} \quad a = \text{ semi major axis.} \tag{7b}$$

2. SECULAR CHANGE OF THE FLATTENING OF THE EARTH

Secular disturbances result from the general formula (2) if one takes into account that the particle cloud is, according to the Riemann-Mach mechanics, a cosmos in evolution (Treder 1972). The radius R of the particle cloud expands either parabolically according to the law

$$R = \sqrt{\frac{2}{3}} ct + \text{const} \tag{8a}$$

or hyperbolically according to the law

$$\frac{4f^2}{k^2}\left(\frac{kR}{fM}+1\right) = \left(\sqrt{\frac{2}{3}} ct + \text{const}\right)^2 . \tag{8b}$$

It follows then from Eq. (2) (neglecting the local mass induction, Eq. (4)) that if

$$m_0^* = m \frac{9}{2} \frac{f}{c^2} \frac{M}{R_0} \quad \left(R_0 = R\left(t = t_0\right)\right) \tag{9}$$

is the value of the instantaneous present inertial mass m_0^*, then the change of the inertial mass is given by

$$m^*(t) = \frac{R_0}{R(t)} m_0^* = \frac{9}{2} \frac{f}{c^2} \frac{M}{R(t)} m , \tag{10}$$

which means according to which model m_0^* of the universe is chosen, either

$$m^* = m \sqrt{\frac{3}{2}} \frac{fM}{3t} = m_0^* \frac{t_0}{t} \tag{10a}$$

or

$$m^* \sim m_0^* \left(\frac{t_0}{t}\right) \tag{10b}$$

respectively.

If the instantaneous value of the inertial mass $m_a^*(A)$ is defined for an arbitrary moment t = A i. e. for an arbitrary value $R_A = R(t = A)$ according to Mach (1933), then the inertial mass of each particle P_a is equal to the heavy mass of the particle

$$m_a^*(A) = m_a \tag{11}$$

(Treder 1972). This means that one gets, instead of a change according to Eq. (10), a reciprocal change of the gravity constant f.

$$f^*(t) = f_0 \frac{R(t)}{R_0} = \frac{2}{9}\frac{c^2}{M} R(t) \tag{11a}$$

(with $f_0 = f =$ present value $(t = t_0)$ of the gravity constant: $R_0 = R(t_0)$): which measure the effective gravity constant determined by the corresponding Newtonian gravitational law.

As however the inertia m_a^* of the bodies P_a does primarily change, all other physical coupling constants k' must change, too, according to the same law, Eq. (11a). If their dynamically measured instantaneous present value $k_0' = k'(t_0)$, then their effective value is given by Treder (1972) at an arbitrary moment as:

$$k'(t) = k_0' \frac{R(t)}{R_0} \tag{12}$$

The relation of the gravity constant to all other physical coupling constants remains therefore the same.

$$\frac{f^*(t)}{k'(t)} = \text{const.} \tag{13}$$

The evolution of the cosmos, Eq. (8), has therefore the consequence that the inertia of all bodies decreases. This is dynamically measured as the reciprocal increase of all other forces acting on that body and the relation of the physical forces acting on the body remain unchanged with each other. The dependence of the gravity constant, Eq. (11a) on the age of the cosmos, as it follows from Mach's principle, differs from that given by Jordan (1955) and from the Dirac hypothesis as introduced into geophysics in two respects,

1. The increase of the gravity constant follows the age of the cosmos from the Mach-Einstein doctrine as $f^* \sim t$ or $f^* \sim t^2$; the Dirac hypothesis, however, results in a reciprocal law, $f \sim t^{-1}$ (Jordan 1955, 1966, Treder 1969).

2. It follows from Mach's principle that the interrelation of the physical forces remains constant (Jeans 1929): $k^{'}/f^{*} \sim t$, while the Dirac hypothesis implies a change in these relations: $k^{'}/f^{*} \sim t^{-1}$.

Due to the latter consequence, the increase of the gravity constant implied by the Mach-Einstein doctrine differs from that in the kinematic relativity theory of Milne (see Heckmann 1968) and from that in Treder's reference tetrad theory (1979, 1971) concerning its cosmological consequences due to the increasing gravity constant. From Milne, a law

$$f^{*}_{Milne} = \frac{c^3}{M} t$$

is especially valid, but all the coupling constants do not increase according to the same law.

It follows from these remarks that cosmological effects present themselves from Mach's principle and from the Mach-Einstein doctrine only in cases where inertia plays a role. The mean Earth radius, for example, does not depend on inertia. The compression of terrestrial material depends only on the ratio of the gravity constant to the material constants, which are primarily determined by the coupling constant k'. As we deal here with pure point mechanics, we suppose that the incompressibility of the material is a consequence of repulsive forces. The relation of these repulsive forces to the gravitational force remains with time then the same. That is why the internal structure of terrestrial material is largely independent of the age of the universe.

Inertial effects appear, in contrast, concerning the shape of the Earth and they are responsible for its flattening. According to Eq. (10) the inertial mass and with it, the inertial force on the particles constituting the Earth's body, decrease due to the expansion of the cosmos. Therefore the flattening of the Earth α (and also of all other celestial bodies) must decrease with time by constant Earth's radius ρ and constant angular velocity of the rotation Ω (Treder 1972).

The flattening α of the Earth is determined by the ratio of the centrifugal force

$$\sim m^{*} \rho^2 \Omega^2 \qquad (14a)$$

to the gravitational force

$$\sim \frac{m f_0}{\rho^2} = \frac{m_0 f_0}{\rho^2} \qquad (14b)$$

and it is therefore given as

$$\alpha = \text{const}\frac{\Omega^2\rho^3}{f_0}\frac{m^*}{m_0} = \text{const}\frac{\Omega^2\rho^3}{f^*}. \tag{15}$$

In Eq. (15), $f^* = \left(m_0/m^*\right)f_0$ is the effective instantaneous value of the gravity constant. If this value increases, α will decrease with the conditions that

$$\rho = \text{const and } \Omega = \text{const}. \tag{15a}$$

The quantitative estimation of this effect is especially easy, if a parabolically expanding cosmos is considered, as in this case the age of the cosmos t is simply the reciprocal value of the Hubble constant H (Heckmann 1968, Einstein 1969):

$$t = \frac{R}{\dot{R}} = H^{-1}. \tag{16}$$

The present age of the cosmos (the reciprocal value of the Hubble constant) is of order of magnitude

$$2.10^{10}a \geqq t_0 = H_0^{-1} > 10^{10}a \tag{17a}$$

where $H_0 = 1/t_0$ is the instantaneous value of the Hubble constant. It is then combined with Eq. (11) to give approximately for the time differences:

$$f^*\left(t_0 + \Delta t\right) = f_0\left[1 + H_0\Delta t\right]. \tag{17}$$

Using Eq. (17)one gets from Eq. (15):

$$\alpha\left(t_0 + \Delta t\right) = \text{const}\frac{\rho^3\Omega^2}{f_0}\left[-H_0\Delta t\right] = \alpha_0\left[1 - H_0\Delta t\right]. \tag{18}$$

It follows from this that the flattening of the Earth some 108 years ago was about 1% greater than it is at present.

$$\alpha\left(t_0 + [1-2].10^8 a\right) \approx \alpha_0\left[1 + 10^{-2}\right]. \tag{18a}$$

This value, according to Munk and MacDonald (1950), is empirically correct.

The concurrent explanation of the decrease of the Earth's flattening α is Darwin's theory of tidal friction (Darwin 1906, 1911) according to which,

$$\Omega_* = \Omega((t)), \quad \dot{\Omega}(t) < 0 \tag{19a}$$

$$\alpha(t + \Delta t) = \text{const}\frac{\rho^3\Omega^2}{f_0}\left[1 + 2\left(\frac{\dot{\Omega}}{\Omega}\right)_0 \Delta t\right] \tag{19b}$$

are valid instead of Eq. (15a). Thus, according to Darwin's theory, the angular velocity Ω of the Earth decrease continuously due to tidal effects. As a consequence, the length of the day τ (measured in inertial or ephemeridal time) increased permanently

$$\tau(t_0 + \Delta t) = \tau_0\left[1 + \frac{\dot{\tau}}{\tau_0}\Delta t\right] = \tau_0\left[-\left(\frac{\dot{\Omega}}{\Omega}\right)\Delta t\right]. \tag{19c}$$

Kelvin has already found arguments against the effects of tidal friction. Jeffreys (1959) has shown that tidal friction depends essentially on the contour of the sea-shelf and correspondingly it can get insignificantly small. Holmberg remarked (see Hoyle 1937) that the Kelvin-resonances of the terrestrial atmosphere would perhaps just compensate the braking effect of the eventual tidal friction as the Kelvin resonances of the Earth's atmosphere stabilize the length of the day. - According to Munk and MacDonald (1960) the tidal friction is too small to explain even one third of the palaeogeophysical result. It may be even so small that its effect can be completely neglected (see later). Mach's principle in its mathematical form (Treder 1972) yields an effect of the geophysically necessary order of magnitude - independent of a possible additional effect of the tidal friction - to which an eventually existing effect of the tidal friction could be added.

3. SECULAR ACCELERATION OF THE MOON

The braking of the rotation of the Earth by tidal friction has been astronomically deduced mainly from the acceleration of the Moon's motion which remains unexplained in all lunar theories of the celestial mechanics according to JC Adams. A similar additional acceleration exists for planetary motions, too, according to Sitter and Spencer-Jones (1956). The relative acceleration $\dot{\omega}$ per ω, where ω is the actual angular velocity of the orbital motion, is the same for the Moon and for the planets. This acceleration has been considered, since Delauney's and Newcomb's work, as virtual and to be a consequence of the change of the length of the day and with it, of the astronomical time scale. The cause of the latter should be Darwin's tidal friction (1911) (see Poincaré, 1911):

A basic difficulty of this tidal friction is that it depends very strongly on the shape of the Earth's surface (especially of the form of the sea-shelves). Thus, it can hardly be computed in a quantitative form, and therefore, by contrast, it is considered as proof for the existence of tidal friction.

It follows, however, from the Mach-Einstein doctrine according to Treder (1972), that there is a realistic acceleration of the Moon and of the planets due to the decrease of their inertia - as their inertial mass is defined as being constant - as a consequence of the increase of the gravity constant f. Kepler's second law reads with Eq. (17) and with an inertial mass of the Moon supposed to be constant:

$$r^2\omega = h = \text{const} \tag{20}$$

and Kepler's third law:

$$r^3\omega^2 = f^* M \tag{21}$$

(M= heavy mass of the central body, i.e. of the Earth). Here r is the radius of the Moon's orbit around the Earth, and ω the orbital angular velocity of the Moon. Equation (20) is the law of angular momentum and Eq. (21) corresponds to the virial law:

$$m^* \upsilon^2 = f \frac{mM}{r} \,, \tag{21a}$$

so that

$$\upsilon^2 = f^* \frac{M}{r} \,. \tag{21b}$$

By substituting Eq. (20) in Eq. (21) one gets:

$$\omega = f^{*2} \frac{M^2}{h^3} = f_0^2 \frac{M^2}{h^3} \frac{R^2(t)}{R_0^2} \,. \tag{22}$$

Equation (22) tells us that the orbital frequency ω increases with the square of the gravity constant f^*

$$\frac{\Delta\omega}{\omega} = \frac{2\Delta f^*}{f^*} = \frac{2\Delta R}{R} \,. \tag{22a}$$

Using again the approximation applied in Eq. (17) one gets:

$$\omega = f_0^2 \frac{M^2}{h^3} \left[1 + 2\left(\frac{\dot{R}}{R} \right)_0 \Delta t \right] . \tag{23}$$

By substituting in Eq. (17a), one gets the formula for the acceleration:

$$\omega = f_0^2 \frac{M^2}{h} - [1 + 2H_0\Delta t] = \omega_0 [1 + 2H_0\Delta t] \tag{24}$$

$$\omega = \omega_q \left[1 \frac{2\Delta t}{t_0} \right]. \tag{24a}$$

The value in Eq. (24a) corresponds to the value resulting for the acceleration from the lunar theory which gives according to Spencer-Jones (1956) an acceleration value $(\dot{\omega}/\omega)_0$ in the interval

$$1.10^{-10} a^{-1} \leq \left(\frac{\dot{\omega}}{\omega} \right)_0 < 2.10^{-10} a^{-1}.$$

In the following section we shall discuss this value in more detail and explain how the realistic acceleration is connected with the hypothesis of an apparent acceleration simulated by the braking of the Earth's rotation.

The equations of motion of an N-body case are given in the Riemann-Mach mechanics by neglecting all local inertial effects (Treder 1972):

$$\frac{d}{dt} m_a^* \dot{x}_a^i = \frac{d}{dt} \left[m_a \frac{R_0}{R(t)} \dot{x}_a^i \right] = f_0 m_a \sum_{b/a}^{n} \frac{m_b x_{ab}^i}{r_{ab}^3}, \tag{25}$$

where m_a, m_b are again the heavy masses. (In the rigorous equation the dependence of the inertial masses m_a^* appears on the gravitational potential of the masses m_b, too). The left side of Eq. (25) explicitly means:

$$-m_a \frac{R_0}{R^2} \dot{R} \dot{x}_a^i + m_a \frac{R_0}{R} \ddot{x}_a^i \ . \tag{25a}$$

In Eq. (25a), the term

$$-m_a \frac{R_0}{R^2} \dot{R} \upsilon_a^i \approx -m_a \frac{R_0}{R} H_0 \upsilon_a^i = -m_a^* H_0 \upsilon_a^i \tag{25b}$$

can be neglected, as the lunar (and planetary) orbit(s) are nearly circular. Thus, we make the approximate hypothesis:

$$\upsilon_a = r\omega \tag{26a}$$

and with this, Eq. (25) becomes

$$-m^* r\omega H + m^* \omega^2 = m f_0 M r^{-2} \tag{26}$$

from which it follows for ω instead of Eq. (22):

$$\omega - 2H_0 \approx f^{*2} \frac{M^2}{h^3} . \tag{26b}$$

Thus, the first term in Eq. (25a) can be neglected in a linear approximation. According to Mach's definition of the inertial mass, Eq. (25) can also be written as

$$\upsilon_a^i = f^* \sum_{ba}^{n} \frac{m_b}{r^3{}_{ab}} x_{ab}^i \ , \tag{27}$$

where f^* is the effective gravity constant, Eq. (11a). If the value of this constant $f^*(0) = f_0$ at time t '= 0 = t - t_0, then we obtain for the time interval of the "history of astronomy" t'

$$f^* = f_0 (1 + H_0 t') ,$$

and the equations of motion, Eq. (27) are:

$$\ddot{x}_a^i = f_0 (1 + H_0 t') \sum_{ba} \frac{m_b}{r_{ab}} x_{ab}^i . \tag{28}$$

Following a line of thought by Jordan (1966), we introduce a conformal transformation of the space-time coordinates by which the Newtonian equations with variable gravity constant become identical with the normal Newtonian equations. By using

$$x^i = (1 + H_0 t') \tag{29a}$$

and

$$\overline{t} = (1 + H_0 t') t' \tag{29b}$$

we get for a linear approximation:

$$d\bar{t} = dt'(1+2H_0t') \qquad (30a)$$

$$\frac{dx^i}{d\bar{t}} = \frac{dt}{d\bar{t}}\frac{d\bar{x}^i}{dt} - (1+H_0t')\frac{dx^i}{dt} + H_0x^i \qquad (30b)$$

and finally

$$\frac{d^2\bar{x}^i}{d\bar{t}^2} = \frac{dt}{d\bar{t}}\frac{d}{dt}\left(\frac{d\bar{x}^i}{d\bar{t}}\right) = (1-3H_0t')\frac{d^2x^i}{dt^2} \qquad (30c)$$

The conformal transformed equations of motion (27) are then:

$$\frac{d^2\bar{x}^i}{d\bar{t}^2} = (1-3H_0t') = f_0\sum_{b+a}\frac{m_b}{r^3_{ab}}x^i_{iab}\,[1+2H_0t'] = f_0\sum_{ba}\frac{m_b}{r^3_{ab}}\bar{x}^i_{ab}\,. \qquad (31)$$

Equations (31) are the Newtonian equations of motion which do not yield any secular acceleration. According to Eq. (29) the unit of the transformed time $\Delta\bar{t}$ does change with respect of the unit Δt of inertial time:

$$\Delta\bar{t} = \Delta t(1+2H_0t') \approx \Delta t'(1+2.10^{-10}a^{-1}t') \qquad (32)$$

(this is the value of the lengthening of the day, given in the literature by Gutenberg, 1957, and by Spencer-Jones, 1957).

Thus, one has two alternative explanations:

1. One supposes that the astronomical scale of time - as far as it is needed for the secular acceleration - is identical with the inertial scale of time (which is e.g. defined by atomic clocks). In this case, the simplified Machian equations of motion (27) are valid instead of the Newtonian equations of motion and these yield a secular acceleration according to Eq. (24). The expression of celestial mechanics concerning secular acceleration is then with $\dot{\varphi} = \omega$

$$\varphi = \varphi_0 + \omega_0 t' + \frac{1}{2}\left(\frac{\dot{\omega}}{\omega}\right)_0 \omega\, t'^2 = \varphi_0 + \omega_0 t' + H_0\omega_0 t'^2\,. \qquad (33)$$

Here

$$b = \frac{1}{2}\left(\frac{\dot{\omega}}{\omega}\right)_0 \omega_0 = H_0 \omega_0 \tag{33a}$$

is the coefficient of secular acceleration.

2. One supposes that the non-corrected Newtonian equations of motion are valid. In this case the secular acceleration results from the difference between the astronomical time scale, Eq. (32) and the inertial time scale. If Ω denotes the angular velocity of the rotation of the Earth, then the secular acceleration of the Moon is:

$$\varphi = \varphi_0 + n\Omega_0 t' + \frac{n}{2}\left|\frac{\dot{\Omega}}{\Omega}\right|_0 \Omega_0 t'^2 = \varphi_0 + n\Omega_0 + bt'^2, \tag{34}$$

where n is the “average motion” of the Moon (related to the time unit) (Poincaré 1911) (see also Schröder and Treder, 1999).

4. CHANGE OF THE EARTH-MOON DISTANCE

According to the secular acceleration of the motion of the Moon (see Section 3) it follows from the conservation of the angular momentum

$$r^2\omega = h = \text{const}$$

that there is a secular decrease of the Earth-Moon distance r:

$$\frac{\dot{r}}{r} = -\frac{1}{2}\frac{\dot{\omega}}{\omega} = -H_0 . \tag{35}$$

Thus, r decreases at present each year by a fraction in the order of magnitude of $(0.5-1.0).10^{-10}r$

$$2\frac{\text{cm}}{\text{a}} \le \frac{\Delta r}{\Delta t} < 4\frac{\text{cm}}{\text{a}} . \tag{35a}$$

Dirac’s hypothesis would require, in contrast, an increase of the distance r by about the same order of magnitude:

$$2\frac{cm}{a} \leq \frac{\Delta r}{\Delta t} < 4\frac{cm}{a}. \tag{36}$$

Darwin's tidal friction would also result in a temporal increase of the distance r, which is, however, in contrast to Eq. (36), negligibly small, as it is given (Poincaré 1911) by

$$\frac{\Delta r}{r} \approx -2\frac{\Theta \Delta \Omega}{mr^2 \omega} \tag{37}$$

(Θ is the inertial moment of the Earth, m = lunar mass).

In principle, the secular change of the distance, Eq. (35) or (36) (and the relative approximately constant distance Earth-Moon, Eq. (37), respectively) can be monitored by laser reflectors supposing a measurement series lasting many years to eliminate periodic changes and random differences in the distance. Such measurements would also enable us to experimentally distinguish between the Mach-Einstein doctrine, Dirac's hypothesis and Darwin's tidal friction. This would achieve the empirical solution of one of the most important cosmological and physical problems, the validity of the Mach-Einstein doctrine.

The amount of the acceleration of the lunar motion is not very accurately known. The corresponding lengthening of the day (it would be a fictitious one if the Mach-Einstein doctrine is valid) is according to Spencer-Jones (1956):

$$\frac{\dot{T}}{T} \approx (1-2).10^{-10} a^{-1},$$

but according to Fricke it is

$$\frac{\dot{T}}{T} \geq 2.10^{-10} a^{-1}.$$

Thus, it is possible in principle that an even smaller amount should be added to the realistic acceleration resulting from Mach's principle which corresponds to a lengthening of the day by tidal fiction. According to Munk and MacDonald (1960), this supplementary value could reach one third of the full acceleration.

If we would in contrast suppose that there is no lengthening of the day, then the acceleration of the Moon's motion would realistically yield according to Eq. (24) the present value of Hubble's constant H_0 - by taking as a basis the linear cosmological model - and with it, the age of the cosmos.

The consequences of the secular changes of the effective gravity constant as they follow from Eq. (11):

$$0.5.10^{-10}a^{-1} < \frac{\dot{f}^*}{f^*} < 1.0.10^{-10}a^{-1}$$

are in accordance with those of celestial mechanics, with the upper limits for the secular change of f deduced from radar-astronomical measurements by Shapiro et al. (1971) in 1971 from the motion of the inner planets. Presupposing that Schwarzschild's statistical line element determines these motions Shapiro found that it must be

$$\left|\frac{\dot{f}}{f}\right| < 4.10^{-10}a^{-1}.$$

It should be remarked that the approximate equations of motion Eq. (25) become invalid if, due to the expansion of the cosmos,

$$f^* \to \infty,\ R \to \infty$$

In such a case the locally induced inertial effects surpass the collective effect of the universe according to Friedlaender und Einstein.

In the limiting case the Lagrangian formula is valid (according to the formula deduced by Treder, 1972), especially for the 2-body problem - i.e. for a Sun-planet system:

$$L^* = f^* \frac{m_1 m_2}{r}\left(1 + \frac{3}{2}\frac{\upsilon^2}{c^2}\right) \qquad \left(r = r_{12}, \upsilon_i = \upsilon^i_{12}\right)$$

with the energy law

$$f^* \frac{m_1 m_2}{r}\left(1 - \frac{3}{2}\frac{\upsilon^2}{c^2}\right) = \text{const}$$

and with the angular momentum law

$$\frac{\partial L^*}{\partial \varphi} = \frac{3}{c^2} f^* m_1 m_2 r\dot{\varphi} = \text{const.}\ .$$

At $r = r(\varphi)$, the equation of motion are, for the 2-body problem:

$$\left(-2 + \frac{3h^2}{c^2}\right)r + \frac{3h^2}{c^2}r'' = 3kr^2 \quad (h, k = \text{const.}).$$

These equations contain only the mutually induced inertia of the bodies P1 and P2 and describe the relative motion of both bodies as postulated as early as 1879 by Neumann in the frame of an inertia-free mechanics. These equations are extremely different from the Kepler equations of motion, as in a very late stage of the development of the universe, celestial mechanics will be significantly different from the Newtonian one.

If the Riemann-Mach dynamics proves to be completely correct in celestial mechanics, then one has to start from the exact Lagrangian function, Eq. (1), together with the definition of mass, Eq. (2), in order to take into account the local deviations of the Riemann-Mach mechanics from the Newtonian one (This is especially true of Einstein's perihelion motion and the change of the effective inertial mass of certain planetoids with the distance from Jupiter for the calculation of their orbits).

A new calculation method by Brosche and Sündermann (1971) of the eventual braking of the rotation of the Earth by tidal friction can be in the order of magnitude given by Munk and MacDonald. Brosche and Sündermann found, however, that according to the development of tides the tidal friction may also result in acceleration of the Earth's rotation and harmonic tides have no effect on the rotation. Therefore they leave open in principle the existence of an effect of tidal friction on the rotation of the Earth.

The secular decrease of the number of days N in a terrestrial year T given by the palaeontologist Wells (1966) and cited by Brosche and Sündermann (1971) appears as a constant length of the day $\tau = \tau_0$ due to the acceleration of the orbital motion of the Earth according to the generally valid formula, Eq. (22):

$$\left(\frac{\dot{\omega}}{\omega}\right)_{\text{Earth}} = 2H_0 .$$

One gets then approximately for the length of the year:

$$T = T_0\left(1 - 2H_0\Delta t\right) > T_0 \qquad \text{for } \Delta t < 0$$

Here T_0 is the length of the terrestrial year today. Thus, using the present length of the year $\tau = \tau_0$ and with $N_0\tau_0 = T_0$:

$$N\tau_0 = T = T_0\left(1 - 2H_0\Delta t\right),$$

that is the number of days N in a year T changes as:

$$N = N_0\left(1 - 2H_0\Delta t\right) > N_0 \qquad \text{for } \Delta t < 0 .$$

The decrease of the speed of the rotation of the Earth as proposed by Wells and others using

$$\frac{\dot{\Omega}}{\Omega} \approx -2H_0 ,$$

results concerning the length of the day in

$$\tau = \tau_0 \left(1 + 2H_0 \Delta t\right) < \tau_0 \qquad \text{for } \Delta t < 0 .$$

For Wells the length of the terrestrial year remains constant, i.e.

$$N\tau = N\tau_0 \left(1 + 2H_0 \Delta t\right) = T_0$$

and therefore the number of days in one year is again the previously given:

$$N = N_0 \left(1 - 2H_0 \Delta t\right) > N_0 \qquad \text{for } \Delta t < 0 .$$

The palaeontological results are therefore explained by the Mach-Einstein doctrine in the same way as by the hypothetically supposed tidal braking of the rotation of the Earth, due to the fact that, here only, the ratio N between length of the year T and length of the day τ plays a role.

5. CONCLUDING REMARK

All treatments around 1900 of the celestial situation reached the conclusion that celestial mechanics based on Newton's gravitational and motion laws can explain all observed facts with three exceptions:

1. The acceleration of Encke's comet, observed by Encke in 1819 and analogous small accelerations of other short periodic comets.
2. The secular acceleration of the lunar motion, discovered in 1853 by Adams (and smaller accelerations in absolute value of the motion of planets).
3. The rotation of the perihelion of Mercury, discovered by Leverrier in 1859 (and similar smaller rotations of the perihelions of other planets).

Whipple then explained the first phenomenon in the framework of classical physics by a model of the comets based on their disintegration. The third effect was explained by Einstein's formula in the framework of the general relativity theory. The same Einsteinian formula follows, however, from inertia-free mechanics, with the help of the Mach-Einstein doctrine (see Part 1).

The secular acceleration of the Moon and of the planets that had not been fully explained hitherto follows now from the Mach-Einstein doctrine as a consequence of the expansion of the cosmos. Thus, the inertia-free mechanics, together with the Mach-Einstein doctrine constitutes the most effective tool of classical celestial mechanics.

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